Distributed Formation Control of Multiagent Systems With Specified Order

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***Abstract*—In this article, we design the controller for multiagent systems such that the agents with nonlinear dynamics can form into arbitrarily given shapes, rotate around the formation centroid at the given angular velocity, track the reference trajectory, and dynamically adjust the formation for obstacle avoidance. In addition, the proposed design ensures that the agents can be positioned with a specified order in the formation. We propose an extended model for the system to facilitate the design. To deal with the order issue, we introduce the *phase weighting parameter* and the *phase penalty flow exchange mechanism* such that the agent's position can be adjusted to the desired order. With the aid of the proposed tools, we first design the formation controller for MAS with a planar unicycle model and then extend it to the three-dimensional space. The controller is designed based on information from the neighbors and is measured in the agents' local reference frame. Moreover, unlike most of the existing works, the initial conditions are not restricted in our approach. Finally, system stability is proved via the Lyapunov theorem and several simulation examples are provided to validate our results.**

***Index Terms*—Cooperative systems, distributed control, formation control, multiagent systems (MASs), nonlinear systems.**

I. Introduction

FORMATION control of multiagent systems (MAS) has wide applications and is extensively studied in recent years due to the rapid development of robotics and UAVs. Generally speaking, the formation control problem is to design the controller which steers an MAS to have the desired geometric pattern and maneuvers the system to perform desired motions cooperatively. Formation control may have different functionalities, such as tracking the reference trajectory for surveillance or navigation [1]–[3], scaling the formation shape for obstacle avoidance or environment adaptation [3]–[5], rotating around a center for data collection or measurement [6], etc. Early researches of formation control consider static formation at fixed desired positions [7]–[9]. Then, the controller is further designed such that the agents can track the trajectory in addition to the static formation such as that in [1]–[3]. Some works further consider the case that the system has circular motion [10], [11] and the formation shape can be adjusted by affine transformation or switching [3]–[5], [12]. In [7], [8], [13], and [14], the problem with arbitrarily given formation shape is discussed, which is more general than only considering a specific formation.

Some researches also consider the task that the agents are positioned with a specific order in the formation. The order relation may be crucial when forming into nonsymmetric shapes or when we want to merge groups of MASs into a larger synthesized structure. For example, when some specific agents are assigned as the connection nodes of two subgroups, the ordered formation structure can help to achieve the task since we can assign the order relation in the formation. Moreover, in some data collection missions, when receiving local data indexed by agent numbers, we can reconstruct the global one by combining them based on the assigned order relation. While the agents with specified order could be advantageous in some scenarios; however, it may sometimes limit freedoms in agents' motions and reduce the flexibility in some tasks. Thus, the assignment of the order relation in the formation depends on the considered tasks. Some papers, e.g., [5], [15], and [16], use interagent potential function and repulsive concept to steer the agents into the splay state (pentagon); however, these methods cannot ensure the formation without all-to-all communication. Rezaee and Abdollahi [10] and Marshall *et al.* [17] proposed a strategy where each agent pursues its front agent cyclically to form into an ordered circular pattern. In such a case, the formation is vulnerable as there is merely one single link for each agent. Though various ordered stable patterns are proved in [17], the design strategy is based on the cyclic communication graph.

On the other hand, communication links and information used for feedback are important topics of formation control of MAS. There are many studies that consider controllers that employ the absolute information and centralized communications of the MAS, which means the global reference frame is needed and the agents can have information of all the other agents, respectively. In [14] and [18], a global reference frame is required to represent the displacement vector $\boldsymbol{h}$

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which describes the vector from the centroid to each agent and aids to achieve the desired formation. This article [18] considers switching directed topologies and utilizes a well-designed time varying $\boldsymbol{h}(t)$ in advance to perform rotating formation; nonetheless, the agent dynamics are linear and the reference trajectory is associated with the control inputs and initial conditions. The result in [14] employs a constant $\boldsymbol{h}$, and some ordered patterns are not allowed due to their assumptions on features of patterns. In [9] and [13], their design allows the formation to achieve a specific order without a global reference frame; however, some restrictions are imposed on initial conditions. They assume close initial headings or initial relative orientations between agents to prevent from converging into incorrect order. Recently, many researches also focus on the distributed communication case where each agent can only get information from their communicable agents and the measurements are in their local reference frames, e.g., [5], [8], [13], [19], and [20]. Instead of general distributed communication, some works consider special types of distributed communication topology, such as the distance-based property or rigidity issue in [3] and [7], the multilayer framework in [21], and the input-based event-triggered communication in [22]. In [4], [5], [18], [23], and [24], the agent's absolute position and global reference frame are needed; in [1] and [25], the desired relative position vectors are defined with respect to the global reference frame; and in [14] and [17], the information is measured in an aligned frame. In practice, to maintain a common global frame is challenging and thus the control law based on each agent's local frame has received much attention as discussed in [2], [8], [9], and [13]. Such design is more involved compared with the global reference frame case.

Based on the above discussions, in this article, we propose a design that drives the MAS to have a formation in exact order without using the global reference frame and global communication, and does not impose restrictions on the initial conditions. More specifically speaking, we design a formation controller of an MAS with homogeneous unicycle type agents in planar and spatial space, respectively. In the preliminary version of this article, we propose a design that achieves formation with assigned order relation in the planar motion [26]. In this article, we extend the control to the three-dimensional (3-D) case with a systematic design approach. The main features and contributions of the proposed design are as follows.

1) We propose an *extended model* based on a new description of the desired formation shape. The extended model helps us formulate the control tasks into a compact design problem which considers arbitrary formation shape, reference trajectory tracking, affine transformation command for environmental adaptation, and specified agent order in the formation.
2) We propose the *phase penalty flow exchange mechanism* and *phase weighting parameter* to prevent the agents from getting stuck in the undesired order and ensure convergence to the correct order in the formation.
3) We design a novel nonlinear formation controller which only uses *distributed communications*. The controller utilizes information from the agent itself and its neighbors, and is measured in each agent's *local reference frame* instead of using a common global frame. Moreover, based on the above design, initial conditions of the agents can be arbitrarily given without imposing additional restrictions.
4) The proposed design is systematically extended to the *3-D space*.

The remainder of this article is organized as follows. Section II provides some preliminaries. In Section III, the formation problem is described and an extended model is proposed. Section IV gives the main results which consist of the phase penalty exchange mechanism, design of the control law, and stability analysis. Section V extends the control to the 3-D space and Section VI provides some simulation examples. Finally, the conclusion is given in Section VII.

II. Preliminaries

We briefly introduce some algebraic graph theory for the communication graph and some existing results of rotational formation control, based on which we develop the extended model for the considered problem.

A. Algebraic Graph Theory

An undirected graph $\mathcal{G} = (V, E)$ consists of a set of nodes V and a set of unordered edges E. An edge connecting V_i, $V_j \in V$ is denoted as $(i, j) \in E$. The adjacency matrix $\boldsymbol{A} = [a_{ij}]$, where $a_{ij} = 1$ for all $(i, j) \in E$, $i \neq j$, and $a_{ij} = 0$ otherwise. The degree matrix $\boldsymbol{D} = [d_{ii}]$ is a diagonal matrix, where the diagonal elements are equal to the number of edges connected to the nodes. The Laplacian matrix $\boldsymbol{L} = [l_{ij}]$ is defined as $\boldsymbol{D} - \boldsymbol{A}$. Define the incidence matrix $\boldsymbol{B} = [b_{ik}]$ where $b_{ik} = 1$ if V_i is the destination of edge-k, $b_{ik} = -1$ if V_i is the source of edge-k, and $b_{ik} = 0$ otherwise. $|\boldsymbol{B}|$ stands for the entrywise absolute value of $\boldsymbol{B}$. A useful property is that $\boldsymbol{L} = \boldsymbol{B}\boldsymbol{B}^T$. Here, we denote $\mathbf{1}$ as the ones vector and it lies in the null space of $\boldsymbol{L}$ and $\boldsymbol{B}^T$. $\boldsymbol{I}_N$ is the N-dimensional identity matrix. In the following contents, we denote N_k the set of nodes whose edges are connected to agent-k, i.e., the neighbors of agent-k, and let $|N_k|$ be the total number of neighbors of agent-k.

B. Formation Control With Circular Motion

The planar unicycle model of homogeneous agents is usually represented as

$$\dot{\boldsymbol{r}}_k = v_0[\cos\varphi_k, \sin\varphi_k]^T, \quad \dot{\varphi}_k = u_k \tag{1}$$

where $\boldsymbol{r}_k = [x_k, y_k]^T \in \mathbb{R}^2$ is the position, $\varphi_k \in \mathbb{R}$ is the heading angle, $v_0 \in \mathbb{R}$ is the velocity, and $u_k \in \mathbb{R}$ is the heading control input which controls the moving direction. The subindex $k \in \{1, 2, \ldots, N\}$ refers to agent-k. Model (1) is controlled by translation velocity v_0 and angular velocity $\dot{\varphi}_k$. To steer N agents with dynamics (1) into an evenly distributed circle, to rotate around the formation centroid at a given angular velocity ϖ_0, and to have the formation centroid tracking the predefined reference trajectory $\boldsymbol{r}_d(t) \in \mathbb{R}^2$, Briñón-Arranz *et al.* [27] proposed a modified model with constant velocity $\bar{v}_0$

$$\dot{\boldsymbol{r}}_k = \bar{v}_0[\cos\theta_k, \sin\theta_k]^T + \dot{\boldsymbol{r}}_d, \quad \dot{\theta}_k = \bar{u}_k \tag{2}$$

where θ_k and $\bar{u}_k$ are the heading angle and the control input of (2), respectively. The physical model of unicycle is (1) and thus, after designing the controller on (2), one needs to find the relation between the physical model (1) and the modified model (2), that is, $\dot{\boldsymbol{r}}_k = v_0[\cos\varphi_k, \sin\varphi_k]^T = \bar{v}_0[\cos\theta_k, \sin\theta_k]^T + \dot{\boldsymbol{r}}_d$. Then, we can obtain the physical control law v_0 and u_k in (1) by the following relations as in [27]: $v_0 = |\dot{\boldsymbol{r}}_k| = |\bar{v}_0[\cos\theta_k, \sin\theta_k]^T + \dot{\boldsymbol{r}}_d|$, and $u_k = [(\ddot{\boldsymbol{r}}_k^T \boldsymbol{R}_{(\pi/2)}\dot{\boldsymbol{r}}_k)/|\dot{\boldsymbol{r}}_k|^2]$, where $\boldsymbol{R}_{(\pi/2)}$ is the 90° counter-clockwise rotation matrix. In this article, to achieve more functionalities, we propose an extended model based on which the controller is designed. The physical control law for the original model can be obtained by similar derivations.

III. Problem Formulation With Extended Model

A. Problem Description

Consider the numbered agents with the unicycle model, and suppose the communication links of the MAS are represented by an undirected connected communication graph $\mathcal{G}$. Given the desired smooth center trajectory $\boldsymbol{r}_d(t) \in \mathbb{R}^2$, the desired formation shape, the smooth reference affine transformation command $\boldsymbol{G}^*(t)$, and the constant angular velocity ϖ_0. Assume that at least one agent can receive $\boldsymbol{r}_d$ and $\boldsymbol{G}^*$. The objective is to design the control law such that the numbered agents can: 1) form into the arbitrarily given shape with a *specified order*; 2) rotate around the centroid at the angular velocity ϖ_0; 3) track the given trajectory $\boldsymbol{r}_d(t)$ with respect to the formation centroid; and 4) transform into various formation shapes based on the reference affine transformation command $\boldsymbol{G}^*(t)$. Here, the reference affine transformation command $\boldsymbol{G}^*(t) \in \mathbb{R}^{2\times 2}$ is used such that the formation can be adjusted into various shapes for environment adaption or obstacle avoidance. As for the communication issue, the designed distributed control law will only use the information from the agent itself and its neighbors, except the information of the desired order and the desired angular velocity for the rotational motion, which are regarded as the control targets and are assumed to be known for all the agents. In addition, the controller does not use measurements in the global reference frame.

Inspired by the modified model (2), we will introduce an extended model which considers arbitrary formation shape, desired order relation, and affine transformation command. The considered problem is then formulated with the extended model.

B. Extended Model and Problem Formulation

For the considered formation task, we start with a unit circular motion

$$\dot{\hat{\boldsymbol{r}}}_k - \dot{\boldsymbol{r}}_d = \varpi_0 \boldsymbol{R}_{(\pi/2)}\big(\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d\big) \tag{3}$$

where $\hat{\boldsymbol{r}}_k$ is the position of agent-k in the unit circular motion and the center of the unit circular motion follows the given trajectory $\boldsymbol{r}_d$. Model (3) served as a virtual structure in our problem. In the following, we first introduce a new description for the formation shape which is rotation invariant as shown in Fig. 1. Then, we derive an extended model to consider arbitrary formation shape and specified order. Given any desired formation shape, say Fig. 1, we calculate the rotation centroid of the desired formation shape. Then, let d_k^* be the length of radius vector from the centroid to agent-k, and let θ_{kj}^* be the desired relative heading phase (angle) between radius vectors of agent-k and agent-j, where the term *phase* implies sign sensitive, that is, $\theta_{kj}^* = -\theta_{jk}^*$. Note that when the agents satisfy the desired phase relation, the assigned order relation is achieved. Based on the descriptions and the unit circular motion, for a desired rotational formation where agent-k is supposed to rotate around $\boldsymbol{r}_d$ with the radius d_k^*, the actual distance from agent-k to $\boldsymbol{r}_d$ should be scaled by a factor d_k^*, i.e., $\boldsymbol{r}_k^* - \boldsymbol{r}_d = d_k^*(\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d)$, where $\boldsymbol{r}_k^*$ is the desired position of agent-k. Moreover, let $\theta_k^*(t)$ be the desired heading angle of agent-k which satisfies $\theta_k^*(t) - \theta_j^*(t) = \theta_{kj}^*$ and $\dot{\theta}_k^* = \varpi_0\ \forall k, j = 1, \dots, N$. Then, when the agent is in the desired phase, the unit vector $\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d$ in the virtual structure can be expressed as $\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d = [\sin\theta_k^*, -\cos\theta_k^*]^T$. If we further consider a reference affine transformation command $\boldsymbol{G}^*$ which is a 2-by-2 matrix that consists of scaling, rotating, or shearing [5], the desired relation between $\boldsymbol{r}_k^*$ and $\hat{\boldsymbol{r}}_k$ becomes

$$\boldsymbol{r}_k^* - \boldsymbol{r}_d = d_k^* \boldsymbol{G}^*\big(\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d\big) = d_k^* \boldsymbol{G}^*\big[\sin\theta_k^*, -\cos\theta_k^*\big]^T. \tag{4}$$

Take time derivatives of (4) and we have

$$\dot{\boldsymbol{r}}_k^* - \dot{\boldsymbol{r}}_d = d_k^* \dot{\boldsymbol{G}}^*\big(\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d\big) + d_k^* \boldsymbol{G}^*\Big(\dot{\hat{\boldsymbol{r}}}_k - \dot{\boldsymbol{r}}_d\Big).$$

Together with (3), we can obtain $\dot{\boldsymbol{r}}_k^* - \dot{\boldsymbol{r}}_d = d_k^*(\varpi_0 \boldsymbol{G}^* - \dot{\boldsymbol{G}}^* \boldsymbol{R}_{[\pi/2]})\boldsymbol{R}_{(\pi/2)}(\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d)$, which is equivalent to

$$\dot{\boldsymbol{r}}_k^* = d_k^*\Big(\varpi_0 \boldsymbol{G}^* - \dot{\boldsymbol{G}}^* \boldsymbol{R}_{(\pi/2)}\Big)\big[\cos\theta_k^*, \sin\theta_k^*\big]^T + \dot{\boldsymbol{r}}_d. \tag{5}$$

The desired position and desired rotational motion of agent-k are described by (4) and (5), respectively. Based on (5), the extended model for the N numbered agents is proposed as

$$\begin{aligned} \dot{\boldsymbol{r}}_k &= d_k^*\big(\varpi_0 \boldsymbol{G}_k - \dot{\boldsymbol{G}}_k \boldsymbol{R}_{(\pi/2)}\big)[\cos\theta_k, \sin\theta_k]^T + \boldsymbol{v}_k \\ \dot{\boldsymbol{v}}_k &= \boldsymbol{\tau}_k,\ \ddot{\boldsymbol{G}}_k = \boldsymbol{T}_k \\ \dot{\theta}_k &= \bar{u}_k \end{aligned} \tag{6}$$

for $k = 1, 2, \dots, N$, where θ_k is the heading angle of agent-k, $\boldsymbol{\tau}_k \in \mathbb{R}^2$ is the translation control that steers $\boldsymbol{v}_k$ to follow the given $\dot{\boldsymbol{r}}_d$, $\boldsymbol{T}_k \in \mathbb{R}^{2\times 2}$ is the control for the affine transformation command such that $\boldsymbol{G}_k$ will track the reference $\boldsymbol{G}^*(t)$, and $\bar{u}_k \in \mathbb{R}$ is the heading control which deals with the moving direction.

With the extended model (6) in mind and the representation of desired formation shape by θ_{kj}^* and d_k^*, the considered

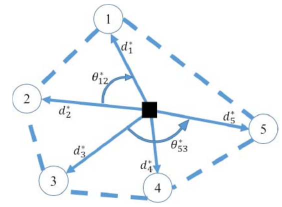


Fig. 1. Desired formation shape with a counter-clockwise order: 1-2-3-4-5. The square at center is the centroid. d_k^* is the length of agent-k's radius vector. θ_{kj}^* is the signed relative angle between agent-k and j, e.g., $\theta_{12}^* < 0$ and $\theta_{53}^* > 0$.

formation problems 1)–4) described in Section III-A can be mathematically formulated as follows. We want to design the control laws $\boldsymbol{\tau}_k$, $\boldsymbol{T}_k$, and $\bar{u}_k$ such that

$$d_k^* \boldsymbol{G}_k [\sin\theta_k, -\cos\theta_k]^T \to (\boldsymbol{r}_k - \boldsymbol{r}_d) \tag{7}$$

$$\theta_{kj} \to \theta_{kj}^*, \quad \bar{u}_k \to \varpi_0, \quad \boldsymbol{v}_k \to \dot{\boldsymbol{r}}_d \tag{8}$$

$$\boldsymbol{G}_k \to \boldsymbol{G}^*, \quad \dot{\boldsymbol{G}}_k \to \dot{\boldsymbol{G}}^* \tag{9}$$

where $\theta_{kj} := \theta_k - \theta_j$ is the relative heading phase between agent-k and agent-j. Note that (7)–(9) imply that the desired position (4) and desired rotational motion (5) with specified order are achieved.

Remark 1: In fact, model (2) can be regarded as a special case of the extended model (6). If the following three special conditions are considered in (6): 1) the affine transformation is not considered, i.e., $\boldsymbol{G}_k = \boldsymbol{I}_2$; 2) all agents can receive $\dot{\boldsymbol{r}}_d$ directly, i.e., $\boldsymbol{v}_k$ can be replaced by $\dot{\boldsymbol{r}}_d$; and 3) only the circular formation is considered, i.e., d_k^* is the radius of a circle. Then, the extended model (6) becomes (2) with $\bar{v}_0 = \varpi_0 d_k^*$.

Remark 2: In some existing works, such as [4] and [5], the information of $\boldsymbol{G}^*$ and $\boldsymbol{r}_d$ is assumed to be globally accessible to all the agents in a common global frame, which is restrictive in some cases. In our considered problem, $\boldsymbol{G}_k$ and $\boldsymbol{v}_k$ are used to track the reference $\boldsymbol{G}^*$ and $\dot{\boldsymbol{r}}_d$, respectively, via the communication links instead of assuming all agents can receive the reference information. In addition, the requirement of a common global frame is also relaxed in this article.

IV. Main Results

In this section, we will introduce the *phase weighting parameter* and the *phase penalty flow exchange mechanism*, which are used to ensure the order relation via the available communication link. Based on the proposed phase penalty flow exchange mechanism and the extended model, the controller is designed with Lyapunov stability analysis.

A. Phase Weighting Parameter and Phase Penalty Flow Exchange Mechanism

Consider a connected communication graph $\mathcal{G}$. For agent-k, we aim to steer $\theta_{kj} \to \theta_{kj}^*$ $\forall j \in N_k, k = 1, 2, \ldots, N$. Let $\tilde{\theta}_{kj} = \theta_{kj} - \theta_{kj}^*$ be the relative heading phase error between agent-k and agent-j. A direct thought to achieve the correct phase (or order) of the agents is to feedback the phase error $\tilde{\theta}_{kj}$ in the control. However, it is possible that the agent will be stuck at a position that forms the desired formation shape but does not have the correct order relation. For example, suppose that $N_k = 1, 2$, and $\tilde{\theta}_{k1} = -\tilde{\theta}_{k2}$, in this case, agent-$k$ may stick to the wrong position due to the cancellation of the phase error. It will be seen more clearly in the proof of control design. To solve the above issue, we define the *phase weighting parameter* $w_k(t)$ of agent-k to avoid the agent from being trapped in an incorrect order. The idea is to utilize the positive time-varying parameter $w_k(t)$ as a weighting of the phase error and thus the phase cancellation effect will not be sustained since the value of $w_k(t)$ will change at the next time instant. To implement this idea, we propose the *phase penalty exchange mechanism* and construct the weighting parameter w_k. Suppose that $w_k(t) > 0$ and is lower bounded by a positive constant $\underline{w}_k$, define the *phase penalty* of agent-k as $\zeta_k = \sum_{j \in N_k} (1 - \cos(\tilde{\theta}_{kj})) \geq 0$, and define the *phase penalty flow* of agent-k as $\Phi_k = (w_k - \underline{w}_k)\ \zeta_k \geq 0$, which is the weighted phase penalty. The phase exchange mechanism is used to distribute the agent's phase penalty flow to its neighbors, that is, for agent-k, $\Phi_k = \sum_{j \in N_k} \phi_{kj}$, where $\phi_{kj} \geq 0$ is the value that agent-k distributes to its neighbor agent-j. Our distributed rule here is to just randomly partition the phase penalty flow Φ_k into $|N_k|$ parts and distribute it to all of its neighbors. Note that ϕ_{kj} and ϕ_{jk} are not necessarily equal and in fact are usually different. By the above distribution process, the *net flow* of agent-k after an exchange iteration is $-\Phi_k + \sum_{j \in N_k} \phi_{jk}$, which includes distributing Φ_k out and receiving the penalty flow from its neighbors. Since the routed out flows are received by the neighbors, we know that the *total net flow* of the MAS is 0 for all time instants, i.e.,

$$\sum_{k=1}^{N} \left(-\Phi_k + \sum_{j \in N_k} \phi_{jk} \right) = 0 \quad \forall t > 0. \tag{10}$$

During the formation process, when $\zeta_k = 0$, agent-k is in the correct order with its neighbors. Thus, agent-k will be removed from the flow exchange mechanism at that moment.

Now, we can choose the value of $w_k(t)$ according to the phase exchange mechanism. Suppose all agents are fixed at some positions and the flow exchanging mechanism is operating. Then, since the formation shape remains unchanged, the sum of phase penalty flow of all agents should remain the same, that is, $(d/dt)(\sum_{k=1}^N \Phi_k) = 0$, which leads to a constraint on $\dot{w}_k$. In addition, $\zeta_k = 0$ since the formation is fixed. Thus, we have $\sum_{k=1}^N \dot{\Phi}_k = \sum_{k=1}^N \dot{w}_k \zeta_k = 0$. To satisfy this equality, we design an update law of w_k based on (10) as follows:

$$\dot{w}_k = \begin{cases} \frac{\delta}{\zeta_k} \left\{ -\Phi_k + \sum_{j \in N_k} \phi_{jk} \right\}, & \text{if } \zeta_k \neq 0 \\ 0, & \text{if } \zeta_k = 0 \end{cases} \tag{11}$$

for $k = 1, \ldots, N$. Here, δ is a positive constant that can be arbitrarily chosen. Note that $w_k(t)$ is lower bounded by $\underline{w}_k$, which can be seen by the fact that once w_k decreases to $\underline{w}_k$, $\Phi_k = (w_k - \underline{w}_k)\zeta_k$ becomes 0 and this leads to $\dot{w}_k \geq 0$ by (11). We will use the phase weighting parameter w_k in the following design to help us deal with the order issue.

B. Notations for the Design

Before presenting the main results, we first introduce some variables and notations which will facilitate the design. Given a desired formation shape and select one agent to be at 0°, then the corresponding degrees of the rest of agents are naturally determined by the desired relative heading phases. Let agent-k's corresponding degree be θ_k^r and define the *reference heading vector* $\boldsymbol{\theta}^r = [\theta_1^r, \ldots, \theta_N^r]^T$. We can see that $\theta_k^r - \theta_j^r = \theta_{kj}^*$, as shown in Fig. 2. With the knowledge of $\boldsymbol{\theta}^r$, we define the *heading shift* $\hat{\boldsymbol{\theta}} := [\theta_1 - \theta_1^r, \ldots, \theta_N - \theta_N^r]^T = \boldsymbol{\theta} - \boldsymbol{\theta}^r \in \mathbb{R}^N$, which satisfies $\hat{\theta}_k - \hat{\theta}_j = \theta_k - \theta_j - (\theta_k^r - \theta_j^r) = \theta_{kj} - \theta_{kj}^* = \tilde{\theta}_{kj}$.

Define $\boldsymbol{e}_{kd} = d_k^* \boldsymbol{G}_k [\sin\theta_k, -\cos\theta_k]^T - (\boldsymbol{r}_k - \boldsymbol{r}_d)$, which is the left-hand side of (7), as the rotational motion error of agent-k. Let $\boldsymbol{e}_{kj} := \boldsymbol{e}_{kd} - \boldsymbol{e}_{jd}$ be the *relative rotational motion error*

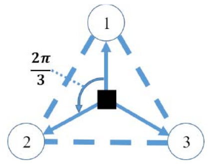


Fig. 2. Consider a regular triangle as the desired shape, and arbitrarily select an agent to be at 0 °, say, agent-1. Then, we have agent-2 at $(2\pi/3)$, agent-3 at $(4\pi/3)$, and the reference heading vector $\boldsymbol{\theta}^r = [0, (2\pi/3), (4\pi/3)]^T$. Moreover, $\theta_2^r - \theta_1^r = \theta_{21}^* = (2\pi/3)$ and so on.

and note that the information of $\boldsymbol{r}_d$ is not needed in $\boldsymbol{e}_{kj}$. Let $\boldsymbol{v}_{kj} = \boldsymbol{v}_k - \boldsymbol{v}_j$ and $\boldsymbol{G}_{kj} = \boldsymbol{G}_k - \boldsymbol{G}_j$ be the relative errors. Denote $\boldsymbol{E}_d = [\boldsymbol{e}_{1d}^T \dots \boldsymbol{e}_{kd}^T \dots \boldsymbol{e}_{Nd}^T]^T \in \mathbb{R}^{2N}$ as the rotational error vector, $\boldsymbol{w} = [w_1 \dots w_k \dots w_N]^T \in \mathbb{R}^N$ as the weighting vector, and $\tilde{\boldsymbol{v}} = [\tilde{\boldsymbol{v}}_1^T \dots \tilde{\boldsymbol{v}}_k^T \dots \tilde{\boldsymbol{v}}_N^T]^T \in \mathbb{R}^{2N}$ as the translation error vector, where $\tilde{\boldsymbol{v}}_k = \boldsymbol{v}_k - \boldsymbol{v}_d$, for $k = 1, 2, \dots, N$, and $\boldsymbol{v}_d = \dot{\boldsymbol{r}}_d$. Note that the phase heading angle θ_k in (2) and (6) is in the global reference frame. In fact, the heading information defined above are based on the global reference frame. We will show how the controller design can be transferred to the local reference frame in the next section.

As N_k denotes the set of the neighbors of agent-k, here we define the extended set $\bar{N}_k$ which additionally includes the reference signal if it is accessible to agent-k. To represent the connection between agent-k and the reference signal, define the diagonal matrix $\boldsymbol{P} = [p_{ii}] \in \mathbb{R}^{N\times N}$, where p_{ii} is 1 if agent-i has access to the reference and 0 otherwise. Let $\bar{\boldsymbol{L}} := (\boldsymbol{L} + \boldsymbol{P})\otimes \boldsymbol{I}_2 \in \mathbb{R}^{2N\times 2N}$ be the extended augmented Laplacian matrix where $\otimes$ denotes the Kronecker product. Since $\boldsymbol{P}$ has at least one positive entry, $\bar{\boldsymbol{L}}$ is positive definite.

C. Design and Stability Analysis of the Controller

We will first design the controller based on the global reference frame in the following theorem and then show how to transfer it to the local reference frame.

Theorem 1: Consider the MAS (6) with randomly given initial positions. Given the connected communication graph $\mathcal{G}$, the smooth reference trajectory $\boldsymbol{r}_d(t)$, the desired angular velocity ϖ_0, and the smooth reference affine transformation command $\boldsymbol{G}^*(t)$, and assume that at least one agent can receive $\boldsymbol{r}_d(t)$ and $\boldsymbol{G}^*(t)$. The control objective (7)–(9) is achieved asymptotically by the control law

$$\bar{u}_k = \varpi_0 - \left\{\alpha \boldsymbol{h}_k^T\left(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}\right) + \beta\sum_{j\in N_k}\left(w_k + w_j\right)\sin\left(\tilde{\theta}_{kj}\right)\right\} \tag{12}$$

$$\boldsymbol{\tau}_k = \frac{1}{|\bar{N}_k|}\left(\sum_{j\in\bar{N}_k}\dot{\boldsymbol{v}}_j\right) - \frac{1}{|\bar{N}_k|}\sum_{j\in\bar{N}_k}\left(\gamma\boldsymbol{v}_{kj} - \alpha\boldsymbol{e}_{kj}\right) \tag{13}$$

$$\boldsymbol{T}_k = \frac{1}{|\bar{N}_k|}\left(\sum_{j\in\bar{N}_k}\ddot{\boldsymbol{G}}_j\right) - \frac{1}{|\bar{N}_k|}\sum_{j\in\bar{N}_k}\left(\lambda\dot{\boldsymbol{G}}_{kj} + \mu\boldsymbol{G}_{kj}\right) \tag{14}$$

for $k = 1, \dots, N$, where $\boldsymbol{h}_k := d_k^*\boldsymbol{G}_k[\cos\theta_k, \sin\theta_k]^T$, w_k is updated by (11), and α, β, γ, λ, and μ, are positive parameters that can be arbitrarily chosen.

Proof: Consider the Lyapunov function candidate

$$V = \frac{\alpha}{2}\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\boldsymbol{E}_d + \beta\boldsymbol{w}^T|\boldsymbol{B}|\left(\boldsymbol{1} - \cos(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})\right) + \frac{1}{2}\tilde{\boldsymbol{v}}^T\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}} \tag{15}$$

where $\cos(\cdot)$ is elementwise. The first term indicates the error between the centroid and $\boldsymbol{r}_d$. The second term penalizes the relative phase errors which are used for the specified order. The last term is for velocity tracking. Let $\Omega \subset \mathbb{R}^{2N}\times\mathbb{R}^N\times\mathbb{R}^{2N}$ be a sufficiently large compact set that contains the origin and the initial state of $(\boldsymbol{E}_d, \hat{\boldsymbol{\theta}}, \tilde{\boldsymbol{v}})$.

We will first show that V monotonically decreases to 0, which proves (7) and (8). Then, (9) will be proved by a consensus algorithm in the existing result. Define $\boldsymbol{h}_k = d_k^*\boldsymbol{G}_k[\cos\theta_k, \sin\theta_k]^T \in \mathbb{R}^2$, and let $\boldsymbol{H} = \mathrm{diag}[\boldsymbol{h}_1, \dots, \boldsymbol{h}_N] \in \mathbb{R}^{2N\times N}$. Note that $\dot{\boldsymbol{e}}_{kd} = (\dot{\theta}_k - \varpi_0)d_k^*\boldsymbol{G}_k[\cos\theta_k, \sin\theta_k]^T - (\boldsymbol{v}_k - \boldsymbol{v}_d)$ and thus $\dot{V}$ can be represented as

$$\begin{aligned}
\dot{V} &= \alpha\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\dot{\boldsymbol{E}}_d + \beta\boldsymbol{w}^T|\boldsymbol{B}|\left(\sin\left(\boldsymbol{B}^T\hat{\boldsymbol{\theta}}\right)\circ\left(\boldsymbol{B}^T\dot{\hat{\boldsymbol{\theta}}}\right)\right) \\
&\quad + \beta\dot{\boldsymbol{w}}^T|\boldsymbol{B}|\left(\boldsymbol{1} - \cos(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})\right) + \tilde{\boldsymbol{v}}^T\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}} \\
&= \alpha\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\boldsymbol{H}(\dot{\boldsymbol{\theta}} - \varpi_0\boldsymbol{1}) + \beta\dot{\boldsymbol{w}}^T|\boldsymbol{B}|\left(\boldsymbol{1} - \cos\left(\boldsymbol{B}^T\hat{\boldsymbol{\theta}}\right)\right) \\
&\quad + \beta\sin\left(\boldsymbol{B}^T\hat{\boldsymbol{\theta}}\right)^T\mathbb{D}\boldsymbol{B}^T\dot{\hat{\boldsymbol{\theta}}} - \alpha\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}} + \left(\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}}\right)^T\tilde{\boldsymbol{v}} \qquad (16) \\
&= \left\{\alpha\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\boldsymbol{H} + \beta\sin\left(\boldsymbol{B}^T\hat{\boldsymbol{\theta}}\right)^T\mathbb{D}\boldsymbol{B}^T\right\}(\dot{\boldsymbol{\theta}} - \varpi_0\boldsymbol{1}) \\
&\quad + \left\{\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}} - \alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d\right\}^T\tilde{\boldsymbol{v}} + \beta\dot{\boldsymbol{w}}^T|\boldsymbol{B}|\left(\boldsymbol{1} - \cos\left(\boldsymbol{B}^T\hat{\boldsymbol{\theta}}\right)\right) \qquad (17) \\
&:= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \qquad (18)
\end{aligned}$$

where $\circ$ denotes the Hadamard product, $\mathbb{D}$ is a diagonal matrix, and $\mathbb{D}_{ii}$ is the ith element of $|\boldsymbol{B}|^T\boldsymbol{w}$. From (16) to (17), we use the fact that the vector $\boldsymbol{1}$ is in the null space of $\boldsymbol{B}^T$. Note that $\dot{V}_1$, $\dot{V}_2$, and $\dot{V}_3$ are related to the heading control, translation velocity control, and weighting update law, respectively. In the following, we will design $\bar{u}_k$ and $\boldsymbol{\tau}_k$ such that $\dot{V}_1 \le 0$ and $\dot{V}_2 \le 0$, and show $\dot{V}_3 = 0$ led by the proposed phase penalty exchange mechanism.

Design the heading control as

$$\dot{\boldsymbol{\theta}} = \varpi_0\boldsymbol{1} - \left\{\alpha\boldsymbol{H}^T\bar{\boldsymbol{L}}\boldsymbol{E}_d + \beta\boldsymbol{B}\mathbb{D}\sin\left(\boldsymbol{B}^T\hat{\boldsymbol{\theta}}\right)\right\} \tag{19}$$

which can be expanded to the heading control (12), where the second term is used to ensure the desired rotational motion and the third term is for the desired relative heading phase θ_{kj}^* $\forall j \in N_k$. By (19), we have $\dot{V}_1 \le 0$. From $\dot{V}_2$, we design

$$\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}} = \alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d - \gamma\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}} \tag{20}$$

such that $\{\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}} - \alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d\}^T = (-\gamma\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}})^T$ and hence $\dot{V}_2 = -\gamma\tilde{\boldsymbol{v}}^T\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}} \le 0$. Equation (20) can be rearranged as $((\boldsymbol{D}+\boldsymbol{P})\otimes\boldsymbol{I}_2)\dot{\tilde{\boldsymbol{v}}} = (A\otimes\boldsymbol{I}_2)\dot{\tilde{\boldsymbol{v}}} + (\alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d - \gamma\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}})$. By definition, $\boldsymbol{D}+\boldsymbol{P}$ is invertible. Using the fact that $(A\otimes B)^{-1} = A^{-1}\otimes B^{-1}$ and $(A\otimes B)(C\otimes D) = (AC)\otimes(BD)$, we have $\dot{\tilde{\boldsymbol{v}}} = ((\boldsymbol{D}+\boldsymbol{P})^{-1}A\otimes\boldsymbol{I}_2)\dot{\tilde{\boldsymbol{v}}} + ((\boldsymbol{D}+\boldsymbol{P})^{-1}\otimes\boldsymbol{I}_2)(\alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d - \gamma\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}})$. The translation velocity control (13) is obtained by expanding $\dot{\tilde{\boldsymbol{v}}}$ above and moving the $\dot{\boldsymbol{v}}_d$ term to the right-hand side of the equation. Note that $\dot{\boldsymbol{v}}_d = \ddot{\boldsymbol{r}}_d$. In (13), the first part is for acceleration consensus and the second part is for velocity consensus. For $\dot{V}_3$, we can express it as $\beta\sum_{k=1}^N \dot{w}_k\zeta_k$ and obtain $\dot{V}_3 = 0$ by (11).

To sum up, V is positive definite, $\dot{V} = -\sum_{k=1}^{N}\{\alpha \boldsymbol{h}_k^T(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}) + \beta\sum_{j\in N_k}(w_k + w_j)\sin(\tilde{\theta}_{kj})\}^2 - \gamma\tilde{\boldsymbol{v}}^T\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}} \leq 0$, and the set Ω is positively invariant. Thus, by invariance principle of nonlinear systems, the state will converge to the set where $\dot{V} = 0$, where

$$\tilde{\boldsymbol{v}}_k = \boldsymbol{0} \tag{21}$$

$$\alpha\boldsymbol{h}_k^T\left(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}\right) + \beta\sum_{j\in N_k}\left(w_k + w_j\right)\sin\left(\tilde{\theta}_{kj}\right) = 0 \tag{22}$$

for $k = 1, \ldots, N$. Equation (21) implies that $\boldsymbol{v}_k = \boldsymbol{v}_d$ and (22) implies that $\bar{u}_k = \varpi_0$ by (12). Then, $\dot{\boldsymbol{e}}_{kd} = \boldsymbol{0}$ for $k = 1, \ldots, N$, and $\dot{\tilde{\theta}}_{kj} = 0\ \forall k \neq j$. Thus, $\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}$, for $k = 1, \ldots, N$, and $\tilde{\theta}_{kj}\ \forall k \neq j$, converge to some constants. Furthermore, note that in (22), the vector $\boldsymbol{h}_k$ keeps moving and the phase weighting parameter w_k keeps updating. Hence, $\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj} \to 0$, for $k = 1, \ldots, N$, and $\tilde{\theta}_{kj} \to 0\ \forall k \neq j$. Moreover, $\bar{\boldsymbol{L}}\boldsymbol{E}_d = \boldsymbol{0}$ leads to $\boldsymbol{e}_{kd} = \boldsymbol{0}$ for $k = 1, \ldots, N$. As a result, (7) and (8) are proved.

The design of $\boldsymbol{T}_k$ for (9) can be obtained by a second-order consensus control with a time-varying reference trajectory. A result from [1] can be directly applied and can be rearranged into (14). ■

Remark 3: In the right-hand side of (12), the design term with parameter α is used to achieve the desired rotational motion, and the term with parameter β is used to guarantee the correct order relation with the aid of the phase weighting parameter w_k. The feedback control in (13) is designed to drive the system to track the given reference trajectory based on the neighbors' information. The design of (14) is a second-order consensus algorithm used to guarantee the convergence of $\boldsymbol{G}_k \to \boldsymbol{G}^*$, $\dot{\boldsymbol{G}}_k \to \dot{\boldsymbol{G}}^*$, which is similar to the result in [1]. Note that in practical implementation of (14), it involves double integration to obtain the signals $\boldsymbol{G}_j$ and $\dot{\boldsymbol{G}}_j$, and there might be some accumulation errors. Thus, for practical implementation, calibration after some time interval may be needed for the construction of the signals in (14).

Remark 4: If we do not consider the time-varying phase weighting parameter $w_k(t)$ in (11) with the penalty flow exchange mechanism, i.e., w_k are constants for $k = 1, 2, \ldots, N$. Then, from the analysis, we still have (22). However, with the constant w_k, we can only ensure $\boldsymbol{e}_{kd} = 0$ and $\sum_{j\in N_k}(w_k + w_j)\sin(\tilde{\theta}_{kj}) = 0$, which cannot guarantee $\tilde{\theta}_{kj} = 0$. For example, suppose $\sin(\tilde{\theta}_{12}) + \sin(\tilde{\theta}_{13}) = 0$ and $\tilde{\theta}_{12}$ and $\tilde{\theta}_{13}$ are constants. Then, there exist infinitely many solutions in addition to the desired case $\tilde{\theta}_{12} = \tilde{\theta}_{13} = 0$, which leaves the ordered formation unaccomplished.

Remark 5: In the existing works [9], [13], formation with a specified order is also considered; however, they need to assume that the initial headings, or, the initial relative orientations, between agents are less than π, and the assigned order relation is implemented based on this assumption. In [17], the order relation can be assigned by cyclically pursuing the previous one agent; however, due to the characteristic of this formation strategy, the communication graph needs to be cyclic. In this article, based on the proposed presentation of the formation, the extended model, and the phase penalty flow exchange mechanism, we do not need additional assumptions on the initial conditions or on the communication graph except the connectivity.

Now, we show how to implement the control law (12)–(14) in each agent's local reference frame. Suppose agent-k has a local frame with counter-clockwise rotation angle ψ_k relative to the common global frame. The variable with superscripted k represents that it is in agent-k's local frame. Let $\boldsymbol{R}(\theta)$ be the rotation matrix of angle θ. Then, the variables in the local frame can be represented by the following relations: $\boldsymbol{e}_{kj}^k = \boldsymbol{R}(-\psi_k)\boldsymbol{e}_{kj}$, $\boldsymbol{v}_{kj}^k = \boldsymbol{R}(-\psi_k)\boldsymbol{v}_{kj} = \boldsymbol{v}_k^k - \boldsymbol{v}_j^k$, $\boldsymbol{\tau}_k^k = \boldsymbol{R}(-\psi_k)\boldsymbol{\tau}_k$, $[\cos\theta_k^k, \sin\theta_k^k]^T = \boldsymbol{R}(-\psi_k)[\cos\theta_k, \sin\theta_k]^T$, and $\boldsymbol{G}_k^k = \boldsymbol{R}(-\psi_k)\boldsymbol{G}_k\boldsymbol{R}(\psi_k)$, $\boldsymbol{T}_k^k = \boldsymbol{R}(-\psi_k)\boldsymbol{T}_k\boldsymbol{R}(\psi_k)$. Note that $\bar{u}_k^k = \bar{u}_k$ since it is a scalar. Define $\boldsymbol{h}_k^k = d_k^*\boldsymbol{G}_k^k[\cos\theta_k^k, \sin\theta_k^k]^T$, which is available in local frame. Then, $\boldsymbol{h}_k^k = d_k^*\boldsymbol{R}(-\psi_k)\boldsymbol{R}(\psi_k)\boldsymbol{G}_k^k\boldsymbol{R}(-\psi_k)[\cos\theta_k, \sin\theta_k]^T = \boldsymbol{R}(-\psi_k)\boldsymbol{h}_k$. Thus, $\boldsymbol{h}_k^T(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}) = \boldsymbol{h}_k^{k^T}(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}^k)$, and (12) is equivalent to

$$\bar{u}_k^k = \varpi_0 - \left\{\alpha\boldsymbol{h}_k^{k^T}\left(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}^k\right) + \beta\sum_{j\in N_k}\left(w_k + w_j\right)\sin\left(\tilde{\theta}_{kj}\right)\right\}. \tag{23}$$

Multiply $\boldsymbol{R}(-\psi_k)$ to both sides of (13) and we have

$$\boldsymbol{\tau}_k^k = \frac{1}{\left|\bar{N}_k\right|}\sum_{j\in\bar{N}_k}\left(\dot{\boldsymbol{v}}_j^k - \gamma\boldsymbol{v}_{kj}^k + \alpha\boldsymbol{e}_{kj}^k\right). \tag{24}$$

Premultiply $\boldsymbol{R}(-\psi_k)$ and postmultiply $\boldsymbol{R}(\psi_k)$ to both sides of (14), then we have

$$\boldsymbol{T}_k^k = \frac{1}{\left|\bar{N}_k\right|}\sum_{j\in\bar{N}_k}\left(\ddot{\boldsymbol{G}}_j^k - \lambda\dot{\boldsymbol{G}}_{kj}^k - \mu\boldsymbol{G}_{kj}^k\right). \tag{25}$$

Hence, the control laws (23)–(25) in agents' local frames are equivalent to (12)–(14) in the global frame.

Corollary 1: For the problem considered in Theorem 1, the control objective (7)–(9) is achieved with the local reference frame control (23)–(25).

Remark 6: Note that the local variables superscripted by k can be obtained without knowing ψ_k. We can check the following relations by the definitions: $\boldsymbol{h}_k^k = d_k^*\boldsymbol{G}_k^k[\cos\theta_k^k, \sin\theta_k^k]^T$, $\dot{\boldsymbol{v}}_j^k = \boldsymbol{R}(-\psi_{kj})\dot{\boldsymbol{v}}_j^j$, $\boldsymbol{G}_{kj}^k = \boldsymbol{G}_k^k - \boldsymbol{R}(-\psi_{kj})\boldsymbol{G}_j^j\boldsymbol{R}(\psi_{kj})$, $\ddot{\boldsymbol{G}}_j^k = \boldsymbol{R}(-\psi_{kj})\ddot{\boldsymbol{G}}_j^j\boldsymbol{R}(\psi_{kj})$, and $\boldsymbol{e}_{kj}^k = d_k^*\boldsymbol{G}_k^k[\sin\theta_k^k, -\cos\theta_k^k]^T - \boldsymbol{R}(-\psi_{kj})d_j^*\boldsymbol{G}_j^j[\sin\theta_j^j, -\cos\theta_j^j]^T - \boldsymbol{r}_{kj}^k$, where $\boldsymbol{r}_{kj}^k$ is the relative displacement from agent-k's viewpoint. A variable subscripted and superscripted by the same agent, say k, is an available local variable in agent-k's reference frame. As a result, with the above relations, the control law (23)–(25) can be realized by measurements in local frames and through neighbors without using ψ_k.

V. EXTENSION TO 3-D SPACE

In this section, we generalize the planar formation control law in Section IV to the spatial space case. Following the design process in the planar case, we first construct the 3-D extended model and then apply the phase penalty flow exchange mechanism. For simplicity, we assume that all agents

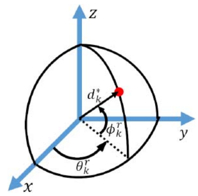


Fig. 3. Constructions of d_k^*, ϕ_k^r, and θ_k^r for a desired 3-D formation.

have aligned z-axis pointing straightly upward. Besides, we consider the scenario that the MAS simultaneously rotates around z-axis at angular velocity α_0 and around x-axis at angular velocity β_0.

Before the derivation of the extended model, we provide the descriptions of the desired 3-D formation. Recall that a point in a sphere coordinate is described by (ρ, ϕ, θ). By this expression, given any desired formation, we denote agent-k's sphere coordinate as $(d_k^*, \phi_k^r, \theta_k^r)$, as shown in Fig. 3. In the following, we derive the extended model based on these notations.

Denote agent-k's desired position by $\boldsymbol{r}_k^* \in \mathbb{R}^3$. Since the desired formation rotates around $\boldsymbol{r}_d$, the desired unit vector after z-rotation and x-rotation becomes

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\phi_k^r \cos\theta_k^r \\ \cos\phi_k^r \sin\theta_k^r \\ \sin\phi_k^r \end{bmatrix}$$

with proper θ and ϕ. Then, refer to the derivation of (4), the desired spatial position $\boldsymbol{r}_k^*$ with $\dot{\boldsymbol{G}}^*$ becomes

$$\boldsymbol{r}_d + d_k^* \boldsymbol{G}^* \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_k^* & -\sin\phi_k^* \\ 0 & \sin\phi_k^* & \cos\phi_k^* \end{bmatrix} \begin{bmatrix} \cos\phi_k^r \cos\theta_k^* \\ \cos\phi_k^r \sin\theta_k^* \\ \sin\phi_k^r \end{bmatrix} \quad (26)$$

for $k = 1, \ldots, N$, where θ_k^* and ϕ_k^* are signals with the following properties: $\dot{\theta}_k^* = \alpha_0$, $\dot{\phi}_k^* = \beta_0$, $\phi_k^* = \phi_j^*$, and $\theta_k^* - \theta_j^* = \theta_k^r - \theta_j^r$, for $k, j = 1, \ldots, N$. Suppose we consider a planar desired formation shape which only rotates around z-axis, i.e., $\phi_k^r = 0$ and $\phi_k^* = 0$, then (26) degenerates to the planar case (4). Now, by derivations similar to those in (5) and (6), the 3-D extended model is

$$\begin{aligned} \dot{\boldsymbol{r}}_k &= \boldsymbol{v}_k + d_k^* \dot{\boldsymbol{G}}_k \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_k & -\sin\phi_k \\ 0 & \sin\phi_k & \cos\phi_k \end{bmatrix} \begin{bmatrix} \cos\phi_k^r \cos\theta_k \\ \cos\phi_k^r \sin\theta_k \\ \sin\phi_k^r \end{bmatrix} \\ &\quad + \alpha_0 d_k^* \boldsymbol{G}_k \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\phi_k & -\cos\phi_k \\ 0 & \cos\phi_k & -\sin\phi_k \end{bmatrix} \begin{bmatrix} \cos\phi_k^r \cos\theta_k \\ \cos\phi_k^r \sin\theta_k \\ \sin\phi_k^r \end{bmatrix} \\ &\quad + \beta_0 d_k^* \boldsymbol{G}_k \cos\phi_k^r \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_k & -\sin\phi_k \\ 0 & \sin\phi_k & \cos\phi_k \end{bmatrix} \begin{bmatrix} -\sin\theta_k \\ \cos\theta_k \\ 0 \end{bmatrix} \\ \dot{\boldsymbol{v}}_k &= \boldsymbol{\tau}_k,\ \ddot{\boldsymbol{G}}_k = \boldsymbol{T}_k,\ \dot{\phi}_k = u_k^\phi,\ \dot{\theta}_k = u_k^\theta \end{aligned} \quad (27)$$

for $k = 1, 2, \ldots, N$, where $\boldsymbol{\tau}_k \in \mathbb{R}^3$, $\boldsymbol{T}_k \in \mathbb{R}^{3\times 3}$, $u_k^\phi \in \mathbb{R}$, and $u_k^\theta \in \mathbb{R}$ are the translation velocity control, affine transformation command control, ϕ-heading control, and θ-heading control, respectively.

With the extended model (27), and given the predefined formation $\{d_k^*, \phi_k^r, \theta_k^r | k = 1, \ldots, N\}$, the desired tracking trajectory $\boldsymbol{r}_d \in \mathbb{R}^3$, the smooth reference affine transformation $\boldsymbol{G}^* \in \mathbb{R}^{3\times 3}$, and constants α_0 and β_0, we want to design the control laws such that

$$\begin{aligned} &\boldsymbol{e}_{kd} \to 0,\quad \phi_{kj} \to 0,\quad \theta_{kj} \to \theta_{kj}^r,\quad u_k^\phi \to \alpha_0,\quad u_k^\theta \to \beta_0 \\ &\boldsymbol{v}_k \to \dot{\boldsymbol{r}}_d, \boldsymbol{G}_k \to \boldsymbol{G}^*, \dot{\boldsymbol{G}}_k \to \dot{\boldsymbol{G}}^* \end{aligned} \quad (28)$$

for $k, j = 1, \ldots, N$, where

$$\boldsymbol{e}_{kd} := d_k^* \boldsymbol{G}_k \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_k & -\sin\phi_k \\ 0 & \sin\phi_k & \cos\phi_k \end{bmatrix} \begin{bmatrix} \cos\phi_k^r \cos\theta_k \\ \cos\phi_k^r \sin\theta_k \\ \sin\phi_k^r \end{bmatrix} - \boldsymbol{r}_{kd}$$

$\boldsymbol{r}_{kd} := \boldsymbol{r}_k - \boldsymbol{r}_d$, $\phi_{kj} := \phi_k - \phi_j$, $\theta_{kj} := \theta_k - \theta_j$, and $\theta_{kj}^r := \theta_k^r - \theta_j^r$. Then, we can derive control laws similar to (12)–(14) as follows:

$$\begin{aligned} u_k^\phi &= \alpha_0 - c_1 \left(\boldsymbol{h}_k^\phi\right)^T \sum_{j\in\bar{N}_k} \boldsymbol{e}_{kj} - c_2 \sum_{j\in N_k} \left(w_k^\phi + w_j^\phi\right) \sin\left(\phi_{kj}\right) \\ u_k^\theta &= \beta_0 - c_1 \left(\boldsymbol{h}_k^\theta\right)^T \sum_{j\in\bar{N}_k} \boldsymbol{e}_{kj} - c_3 \sum_{j\in N_k} \left(w_k^\theta + w_j^\theta\right) \sin\left(\tilde{\theta}_{kj}\right) \\ \boldsymbol{T}_k &= \frac{1}{|\bar{N}_k|} \left(\sum_{j\in\bar{N}_k} \ddot{\boldsymbol{G}}_j\right) - \frac{1}{|\bar{N}_k|} \sum_{j\in\bar{N}_k} \left(c_5 \dot{\boldsymbol{G}}_{kj} + c_6 \boldsymbol{G}_{kj}\right) \\ \boldsymbol{\tau}_k &= \frac{1}{|\bar{N}_k|} \left(\sum_{j\in\bar{N}_k} \dot{\boldsymbol{v}}_j\right) - \frac{1}{|\bar{N}_k|} \sum_{j\in\bar{N}_k} \left(c_4 \boldsymbol{v}_{kj} - c_1 \boldsymbol{e}_{kj}\right) \end{aligned} \quad (29)$$

where $\boldsymbol{e}_{kj} = \boldsymbol{e}_{kd} - \boldsymbol{e}_{jd}$, $\tilde{\theta}_{kj} = \theta_{kj} - \theta_{kj}^r$, w_k^ϕ, and w_k^θ are obtained by applying two independent phase penalty flow exchange mechanisms to phase ϕ and θ, respectively, $c_i > 0, i = 1, \ldots, 6$, are design constants, and

$$\begin{aligned} \boldsymbol{h}_k^\phi &= d_k^* \boldsymbol{G}_k \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\phi_k & -\cos\phi_k \\ 0 & \cos\phi_k & -\sin\phi_k \end{bmatrix} \begin{bmatrix} \cos\phi_k^r \cos\theta_k \\ \cos\phi_k^r \sin\theta_k \\ \sin\phi_k^r \end{bmatrix} \\ \boldsymbol{h}_k^\theta &= d_k^* \boldsymbol{G}_k \cos\phi_k^r \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_k & -\sin\phi_k \\ 0 & \sin\phi_k & \cos\phi_k \end{bmatrix} \begin{bmatrix} -\sin\theta_k \\ \cos\theta_k \\ 0 \end{bmatrix}. \end{aligned}$$

To prove that (28) is achieved with control law (29), consider the Lyapunov function candidate

$$\begin{aligned} V = \frac{c_1}{2} \boldsymbol{E}_d^T \bar{\boldsymbol{L}} \boldsymbol{E}_d &+ c_2 \left(\boldsymbol{w}^\phi\right)^T |\boldsymbol{B}| \left(\boldsymbol{1} - \cos\left(\boldsymbol{B}^T \hat{\boldsymbol{\phi}}\right)\right) \\ &+ c_3 \left(\boldsymbol{w}^\theta\right)^T |\boldsymbol{B}| \left(\boldsymbol{1} - \cos\left(\boldsymbol{B}^T \hat{\boldsymbol{\theta}}\right)\right) + \frac{1}{2} \tilde{\boldsymbol{v}}^T \bar{\boldsymbol{L}} \tilde{\boldsymbol{v}} \end{aligned} \quad (30)$$

where $\boldsymbol{E}_d = [\boldsymbol{e}_{1d}^T, \ldots, \boldsymbol{e}_{Nd}^T]^T \in \mathbb{R}^{3N}$, $\bar{\boldsymbol{L}} = (\boldsymbol{L} + \boldsymbol{P}) \otimes \boldsymbol{I}_3 \in \mathbb{R}^{3N\times 3N}$, $\boldsymbol{w}^\phi = [w_1^\phi, \ldots, w_N^\phi]^T \in \mathbb{R}^N$, $\hat{\boldsymbol{\phi}} = [\phi_1 - \phi_1^r, \ldots, \phi_N - \phi_N^r]^T \in \mathbb{R}^N$, $\boldsymbol{w}^\theta = [w_1^\theta, \ldots, w_N^\theta]^T \in \mathbb{R}^N$, and $\hat{\boldsymbol{\theta}} = [\theta_1 - \theta_1^r, \ldots, \theta_N - \theta_N^r]^T \in \mathbb{R}^N$. The stability analysis and the local reference frame control can be shown by similar discussions as in Theorem 1 and Corollary 1.

Remark 7: Dong and Hu [18] proposed a distributed formation controller that can deal with switching topologies and high-order agent dynamics with linear model. The proposed approach considers a time-varying vector $h(t)$ used for the representation of the formation and can also achieve rotational motion with specified order by appropriately constructing $h(t)$. In addition to the linear model considered in [18], the main differences compared with our results are that the controller

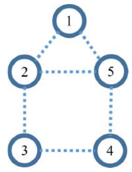


Fig. 4. Desired formation shape.

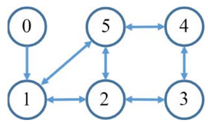


Fig. 5. Communication links.

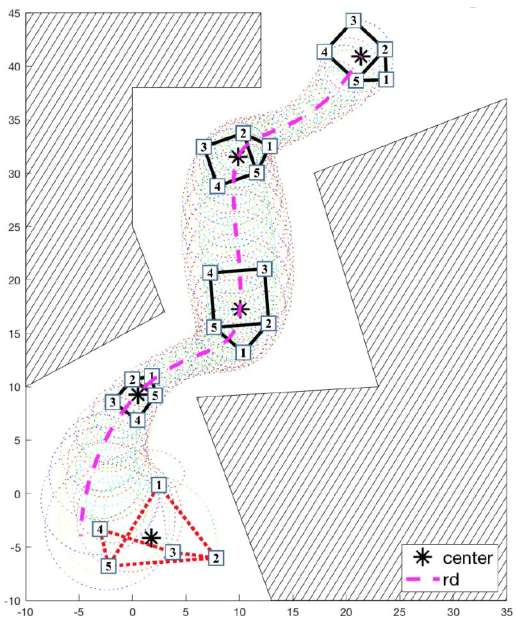


Fig. 6. Ordered formation with environment adaptation.

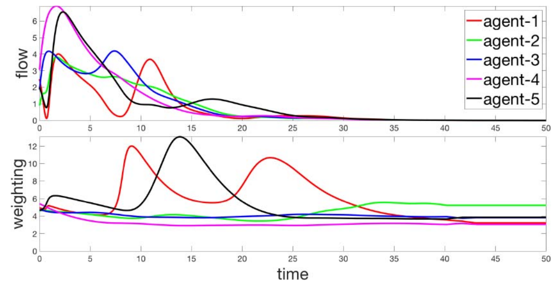


Fig. 7. Phase penalty flow Φ_k and phase weighting parameter w_k.

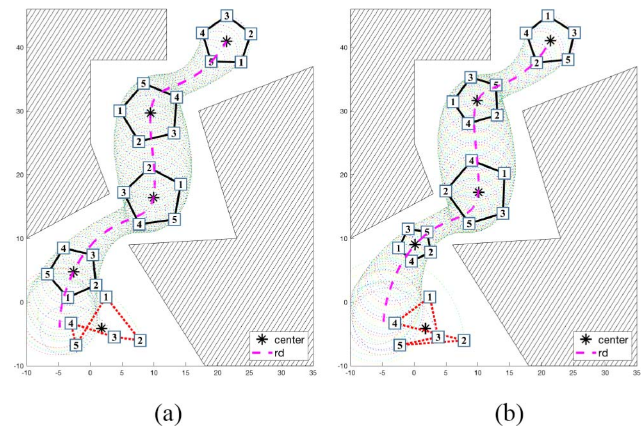


Fig. 8. (a) Ordered formation. (b) Unordered formation by [5].

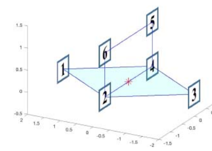


Fig. 9. Desired 3-D formation.

in [18] is implemented in a global reference frame and the reference trajectory in [18] is related to some conditions associated with the control inputs and the initial conditions of the agents. In this article, the controller can be implemented in the local reference frame and the reference trajectory can be previously given such that the system can track the trajectory while keeping the rotational formation. On the other hand, the convergence of the controller proposed in [18] is exponential and is quite satisfactory, whereas the convergence of our controller is asymptotic.

VI. Simulation Results

In this section, we first consider an example with five agents in planar motion. To show the result of achieving a specified order, we compare the unordered one in [5] for the circular formation case. Then, two examples of the 3-D space case are provided. Consider the desired formation and communication graph given in Figs. 4 and 5, respectively, where the index 0 is the reference signal that gives information, such as $\boldsymbol{r}_d$ and $\boldsymbol{G}^*$. The parameters for simulation are as follows: $d_1^* = 3.2, d_2^* = 2.3, d_3^* = 3.4, d_4^* = 3.4, d_5^* = 2.3$, $\boldsymbol{\theta}^r = [1.6, 2.6, -2.2, -1, 0.5]^T$, $\boldsymbol{r}_d = [0.1t - 3 - 3\cos(0.04t - 0.9), 0.2t - 2 + 2\cos(0.06t + 3.1)]^T$, and $\boldsymbol{G}^* = (1 + 0.5\cos(0.05t))\boldsymbol{I}_2$. The initial positions and headings of the five agents are randomly given by $[2.5, 0.8]^T$, $[7.8, -6]^T$, $[3.8, -5.5]^T$, $[-3, -3.3]^T$, $[-2.2, -6.8]^T$, and $-0.4, 1.4, 2.3, -1.3, 0.1$, respectively. Then, by control laws (23)–(25) with $\alpha = 1$, $\beta = 2$, $\gamma = 5$, $\lambda = 2$, and $\mu = 1$, the MAS achieves ordered formation while the formation center tracks $\boldsymbol{r}_d$, and can be dynamically adjusted into smaller scale to safely pass the slits by the appropriately given $\boldsymbol{G}^*$, as shown in Fig. 6. Moreover, the variations of phase penalty flows and phase weighting parameters are depicted in Fig. 7. It can be seen that all flows reduce to 0 and the weightings converge to constants, which imply that the specified order requirement is achieved.

In Fig. 8, we consider the circular motion case, i.e., $d_k^* = 4$ for $k = 1, \ldots, 5$, $\boldsymbol{\theta}^r = [0, 2\pi/5, 4\pi/5, 6\pi/5, 8\pi/5]^T$, and compare our result with [5]. Fig. 8(a) shows that our approach achieves the circular formation with specified order 1-2-3-4-5. Whereas in Fig. 8(b), the result of [5] does not control the order and ends up with 1-3-5-2-4.

For the 3-D space case, consider the MAS with six agents and the desired 3-D formation given in Fig. 9, where

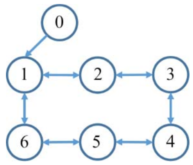


Fig. 10. Communication links.

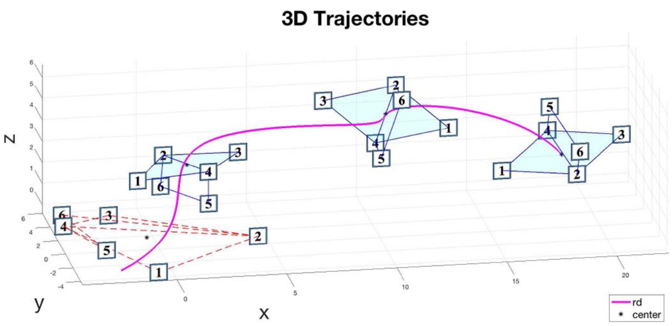
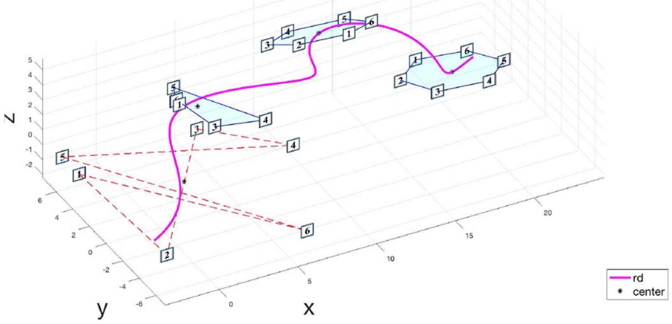
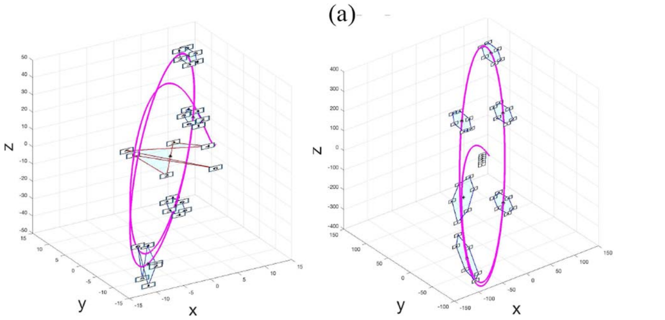


Fig. 11. 3-D ordered formation with rotation and tracking.



(a)



(b)

Fig. 12. Comparisons with the design in [18]. (a) Formation with our proposed controller. (b) Formation with the controller in [18] and different initial positions.

$d_1^* = 2$, $d_2^* = 1$, $d_3^* = 2$, $d_4^* = 1$, $d_5^* = \sqrt{2}$, $d_6^* = \sqrt{2}$, $\boldsymbol{\theta}^r = [0, 0, 0, 0, (\pi/4), (\pi/4)]^T$, and $\boldsymbol{\phi}^r = [(\pi/2), \pi, (3\pi/2), 0, 0, \pi]^T$. The communication graph is shown in Fig. 10. The initial positions of the agents are randomly given in the xy-plane. By our spatial control law (29), the MAS achieves 3-D ordered formation and the desired rotational motion as shown in Fig. 11, which validates our results.

In Fig. 12, we provide another simulation scenario where the desired formation shape is a hexagon and the initial positions are randomly selected as $[-3, 4, 0]^T$, $[-2.5, -5.6, 0]^T$, $[5, 5, 0]^T$, $[9, 1, 0]^T$, $[-3, 6, 0]^T$, and $[5.6, -7.1, 0]^T$ from agent-1 to agent-6, respectively. Fig. 12(b) is the simulation results from the example in [18], where the design parameters are the same as that in [18] except the initial positions. The left-hand side of Fig. 12(b) has the same initial positions as that in Fig. 12(a) while the right-hand side of Fig. 12(b) has initial positions $[1, 2, 3]^T$, $[4, 5, 6]^T$, $[7, 8, 9]^T$, $[10, 11, 12]^T$, $[13, 14, 15]^T$, and $[16, 17, 18]^T$, which form a line. It can be observed that both designs can achieve formation with specified order and rotational motion with given angular velocity. As we discussed in Remark 7, in addition to the requirement of the global reference frame, the reference trajectories of [18] depend on the initial positions, while in our design the reference trajectory can be given independently. Note that the result in [18] can handle the switching topologies with suitable dwell time; however, for comparison, in Fig. 12, we only consider a static communication graph as shown in Fig. 10 without the node-0.

VII. Conclusion

In this article, we designed a formation controller such that the agents can have specified order in the formation. The formation shape and initial conditions can be arbitrarily given and the system can track a reference trajectory. The proposed control laws only measure local information from the neighbors and are designed in each agent's local frame. As a result, the MAS can achieve formation with specified order, rotate around the centroid, track a desired trajectory, and transform into various shapes. We proposed an extended model which additionally considers arbitrary formation shape, phase error, and affine transformation. Moreover, the phase weighting parameter and phase penalty flow exchange mechanism are introduced to help avoid being trapped in the incorrect order. Furthermore, we do not impose restrictions on initial conditions or assume specific communication graphs except the undirected connected graph assumption. In most of the existing results, the reference trajectory and reference affine transformation command are assumed to be globally accessible, whereas in our design, the global information is not required and thus preserves the ability of on-site changing. In future work, MAS with communication delays, directed communication graphs, switching topologies, constrained inputs, and failure agents or new coming agents are research directions of interest. In addition, the generation of the reference affine transformation command $G^*(t)$ is also a topic that can be further studied.

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