Ordered formation control and affine transformation of Multi-Agent Systems without global reference frame

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National Taiwan University

Outline

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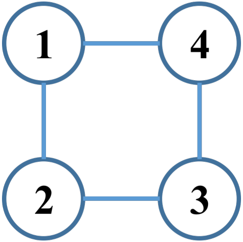
Introduction - formation control

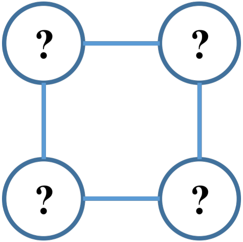
Outline

Introduction - objective

- Ordered formation



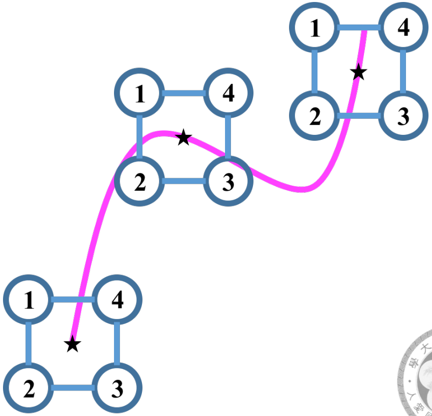
(a) Order



(b) Disorder

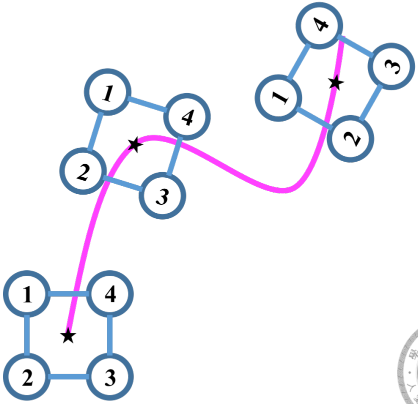
Introduction - objective

- Ordered formation
- Tracking



Introduction - objective

- Ordered formation
- Tracking
- Rotating



Introduction - objective

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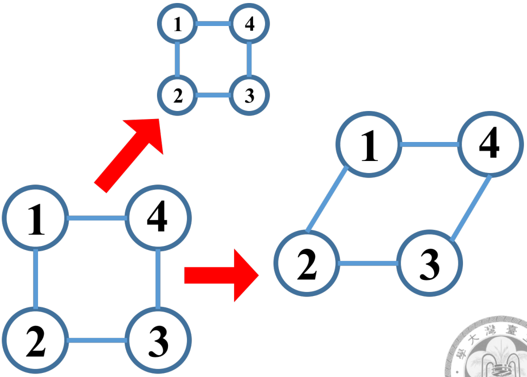
- Transforming



Introduction - objective

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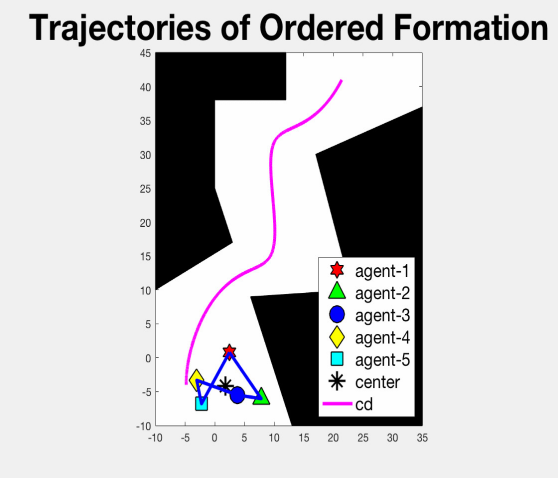
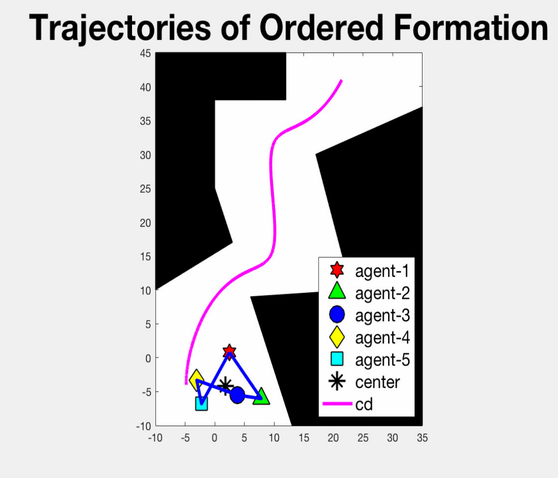


Introduction - objective

- Ordered formation
- Tracking
- Rotating
- Transforming

- Distributed Communications
- Nonholonomic Constraints



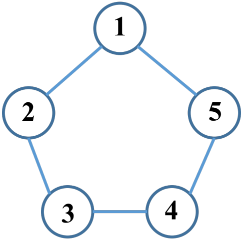
Outline

Introduction - importance

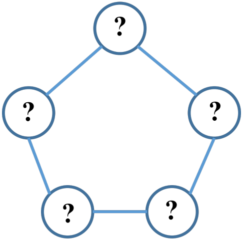
- Ordered formation
- Tracking
- Rotating
- Transforming

Introduction - importance

- Ordered formation
  sensor fusion
- Tracking
- Rotating
- Transforming



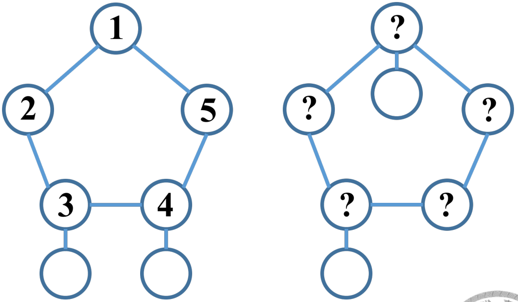
(a) order



(b) disorder

Introduction - importance

- Ordered formation
  sensor fusion
  group synthesis
- Tracking
- Rotating
- Transforming



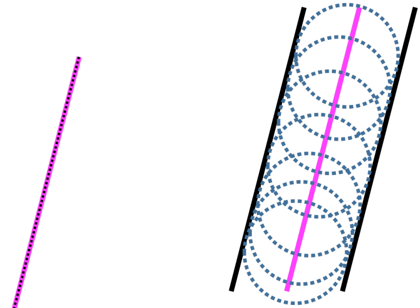
(a) order (b) disorder

Introduction - importance

- Ordered formation
  sensor fusion
  group synthesis
- Tracking
  common task
- Rotating
- Transforming

Introduction - importance

- Ordered formation
  sensor fusion
  group synthesis
- Tracking
  common task
- Rotating
  area extension
  bias reduction
- Transforming

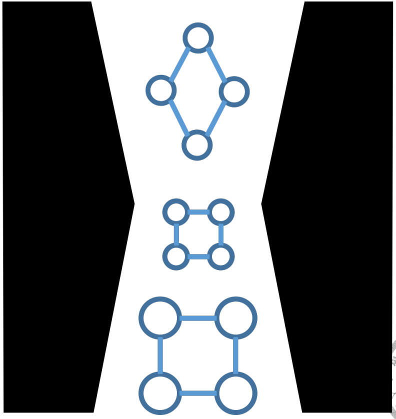


(a) only tracking

(b) tracking + rotating

Introduction - importance

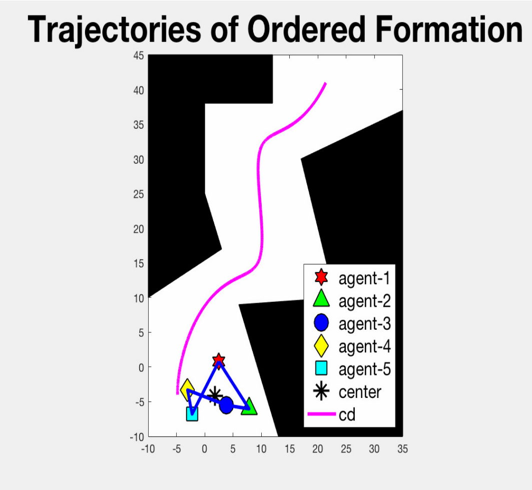
- Ordered formation
  sensor fusion
  group synthesis
- Tracking
  common task
- Rotating
  area extension
  bias reduction
- Transforming
  adaptation



Outline

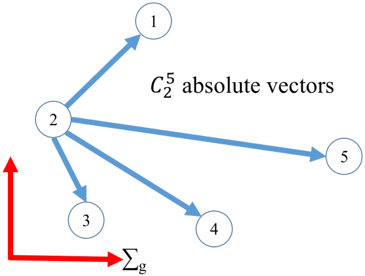
Problem Formulation

- Three main factors:
  - communications
  - desired shape
  - agents

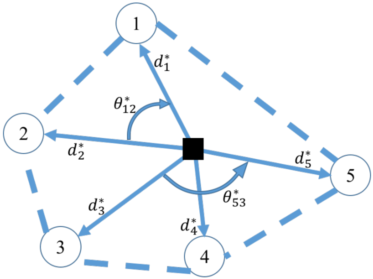


Problem Formulation - desired formation shape

Most papers:



Our thought:



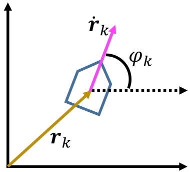
- pros: frame-invariant
- cons: centroid determination

Problem Formulation - dynamic model

- Recall nonholonomic constraint:

$$\dot{\boldsymbol{r}}_k = \nu_k \begin{bmatrix} \cos\varphi_k \\ \sin\varphi_k \end{bmatrix}$$

$$\dot{\varphi}_k = u_k$$



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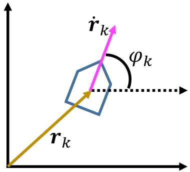

- Extended Model:

$$\dot{\boldsymbol{r}}_k = d_k^*(\varpi \boldsymbol{G}_k - \dot{\boldsymbol{G}}_k \boldsymbol{R}_{\frac{\pi}{2}}) \begin{bmatrix} \cos\theta_k \\ \sin\theta_k \end{bmatrix} + \boldsymbol{v}_k$$
$$\dot{\theta}_k = \bar{u}_k, \dot{\boldsymbol{v}}_k = \boldsymbol{\tau}_k$$

Note: $\boldsymbol{G}_k \in \mathbb{R}^{2\times 2}$ is a linear transformation matrix

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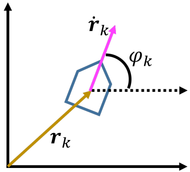

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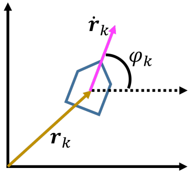

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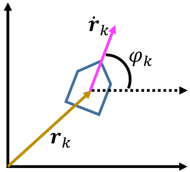

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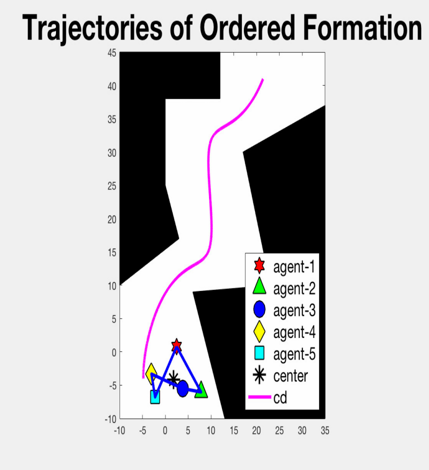
Problem Formulation - proposed problem

Given Information: (M agents)

- desired shape $\{d_k^*, \theta_{kj}^*\}$
- tracking reference $\boldsymbol{r}_d$
- angular velocity ϖ_0
- transformation reference $\boldsymbol{G}^*$

Objectives:

- $\theta_{kj} \to \theta_{kj}^*$
- $\boldsymbol{r}_k \to d_k^* \boldsymbol{G}_k [\sin\theta_k, -\cos\theta_k]^T + \boldsymbol{r}_d$
- $\bar{u}_k \to \varpi_0$
- $\boldsymbol{G}_k \to \boldsymbol{G}^*$



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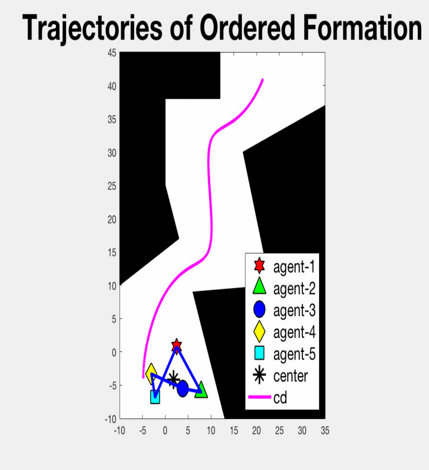
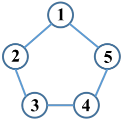


Outline

Mechanism - motivation

- Desired formation shape:



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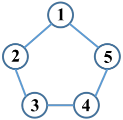

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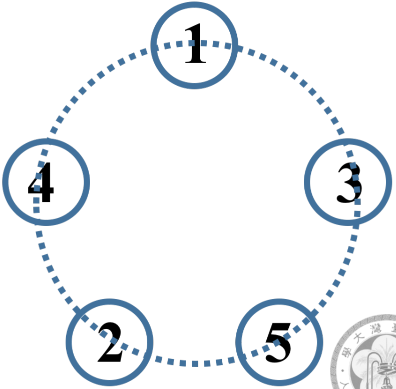

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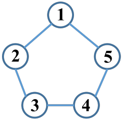

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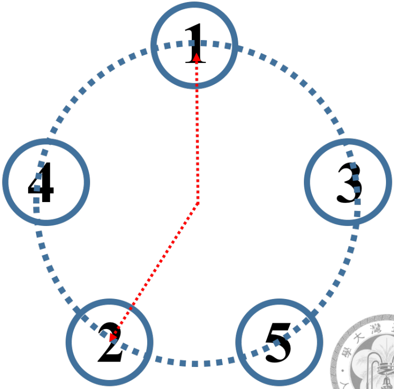

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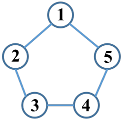

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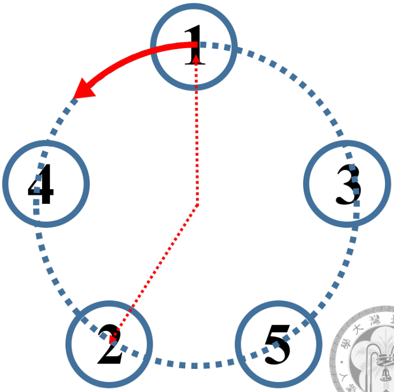

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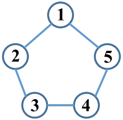

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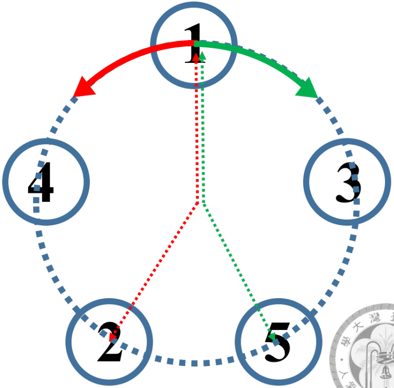

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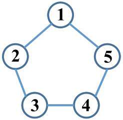

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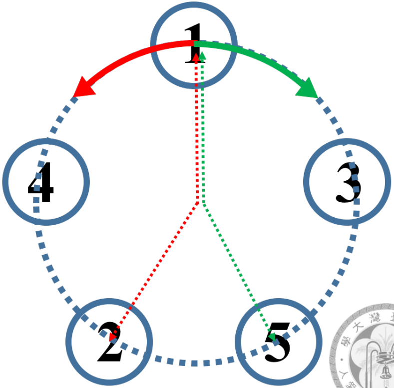
- Stuck in incorrect order



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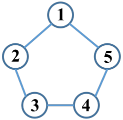

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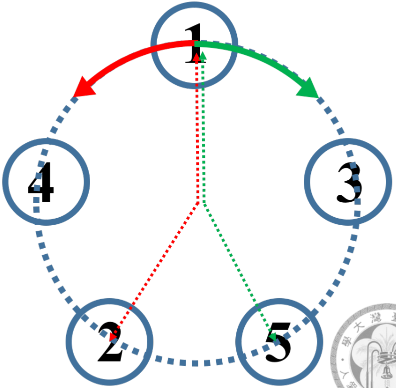
$\Rightarrow$ Time varying weight to resolve cancellation



Outline

Mechanism - design I

Definition

- phase penalty ζ_k: $\sum_{j \in N_k}(1 - \cos(\theta_{kj} - \theta^*_{kj}))$
- weighting parameter w_k: $w_k(t) \geq \underline{w}_k > 0$
- phase penalty flow Φ_k: $(w_k - \underline{w}_k)\zeta_k$

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Phase Penalty Flow Exchange Mechanism

Each agent distributes out its Φ_k to neighbors.

Example

Denote ϕ_{kj} as the amount agent-k distributes to agent-j, then $\Phi_k = \sum_{j \in N_k} \phi_{kj}$

Mechanism - design II

- How to design the update law of w_k?
  Consider a fixed formation with exchange mechanism.

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  - Net flow: $-\Phi_k + \sum_{j \in N_k} \phi_{jk}$
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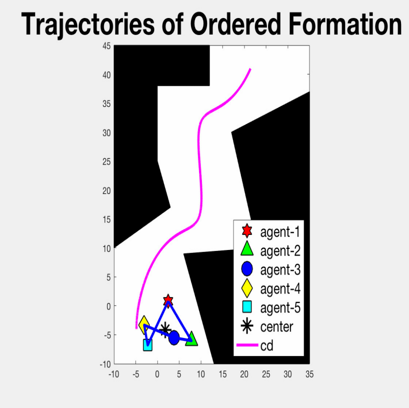
  - $\frac{d}{dt}(\sum_{k=1}^{M} \Phi_k) = 0 = \sum_{k=1}^{M} \dot{w}_k \zeta_k$

$\Rightarrow$ $\dot{w}_k = \frac{c}{\zeta_k}(-\Phi_k + \sum_{j \in N_k} \phi_{jk})$, c is a constant

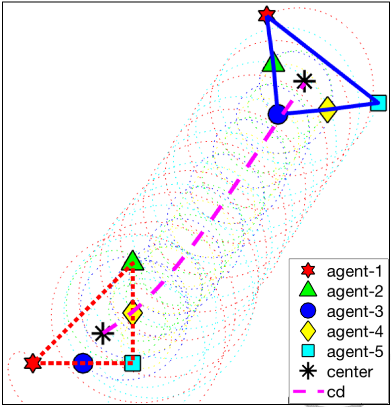
Outline

Simulation Results

- Overall result



- Paradigm - order issue



Outline

Conclusion

- Design a control law such that the MAS tracks, rotates, adapts, and forms in order.
- Propose "phase penalty flow exchange mechanism" to achieve ordered formation.
- Provide stability analysis and some simulation results.
- Future work: extend to spatial case (3D)