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Ordered formation control and affine transformation of Multi-Agent Systems without global reference frame

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Abstract— **The purpose of this paper is to design a control law such that the multi-agent system can form into arbitrary shape, rotate around the centroid which tracks a given trajectory, and further adjust the formation into various shapes based on the affine transformation command. Moreover, the specified order between agents is crucial in some tasks, and hence ordered formation is addressed in our approach. The information for controller is measured locally from the neighbors and is in the local reference frames. To facilitate the goals, we propose an extended model and introduce the *penalty flow exchanging mechanism* which deals with the ordered formation. The control law is derived based on stability analysis, and a simulation example is provided to validate our results.**

I. INTRODUCTION

The researches about Multi-Agent control Systems (MAS) have attracted significant attention for its wide applications in the past two decades. It covers not only control engineering but also consensus algorithm [1], [2], algebraic graph theory [3], [4], and matrix theory [4]–[6]. Formation control, as one of the popular topics in MAS, is to design control laws which steer MAS to form into the desired topology cooperatively. Formation with fixed desired positions is considered in [3], [6], [7]. Nevertheless, most of the applications require movements. Thus, the authors in [2], [4], [8] improve the static formation problem such that the MAS can track a given trajectory in addition to form into a predefined shape. In the meantime, some tasks also require the MAS to rotate at a given angular velocity as in [9]–[11].

Besides, the authors in [4], [10], [12] allow the formation shape to be adjusted by an affine transformation command. Such command is crucial when adapting to the environment such as avoiding obstacles, *e.g.*, [4], [12]. Furthermore, [5], [9] focus on circular formation, and the authors in [10], [12] extend the case to the affine transformation of circle. While a more general problem which considers arbitrary formation shape is discusseded in [3], [6], [11], [13], the order between agents becomes an issue. To solve the order problem, in [5], [14], a pursuit formation strategy is used where only single neighbor is considered; in [11], [15], a common global frame is needed, and [7], [13], [16] assume all-to-all communications or restrictions on initial conditions.

Communication links of the MAS is another topic. The authors in [16] assume centralized communications. Recently, more researches focus on distributed communications case, which means that each agent can get information from the initially communicable agents in the formation process, *e.g.,* [6], [12], [13]. Instead of general distributed cases, some papers consider only specific types of distributed communication topology due to the distance-based property or rigidity issue, such as [3], [4]. Still another topic is the requirement of a common global reference frame. In [10], [12], [15], global frame is required for agents' absolute positions; in [2], [17], the desired relative position vectors are defined with respect to the global frame; in [11], [14], information is measured in the aligned frame. However, to maintain a common global frame is challenging. Thus, the control law based on each agent's local frame is desired and discussed in [6]–[8], [13].

In this paper, we propose an extended model to facilitate the design, and derive the control law in the *local frames* and with *distributed communications*. Such control law steers the numbered MAS with *random initial conditions* to form into an *arbitrary shape* with a *specified order*, and *rotate* around the centroid which *tracks a given trajectory*. In addition, the formation shape can be changed to adapt to the environment by the *affine transformation command*. A related work to ours is [12], where they particularly consider the unordered circular formation, assume the desired trajectory is accessible to all agents, and require a common global frame. In summary, the main contributions of this paper are that we propose an extended model which additionally considers arbitrary formation shape and affine transformation commands, and design a novel control law that achieves ordered formation without common global frame and initial condition restrictions.

This paper is organized as follows: In Section II, we depict our problem and some existing results. In Section III, our extended model is proposed and the problem is re-formulated with the extended model. In Section IV, we introduce the phase penalty exchanging mechanism and derive the control law with stability analysis. In Section V, an example is illustrated and the conclusion is given in Section VI.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

The objective of this paper is to make the numbered MAS form into an arbitrarily given shape and in *specified order*, while rotating at the given angular velocity ϖ_0 around the

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centroid which tracks the predefined trajectory $\boldsymbol{r}_d(t) \in \mathbb{R}^2$. Furthermore, the formation can further change into various shapes by introducing a reference affine transformation command $\boldsymbol{G}^*(t) \in \mathbb{R}^{2\times 2}$. As for the communications, the information is measured locally from neighbors and in each agent's local frame. To illustrate the concept of specified order, take a desired formation, pentagon, as an example. The one with agent's number order 1-2-3-4-5 is different from the one with 1-4-2-5-3 or 1-5-4-3-2. The order issue is crucial when forming into non-symmetric shapes or sticking groups of MASs into larger synthesized structure.

In the follows, we briefly introduce some Algebraic Graph Theory for communication graph, and then discuss the dynamic models of agents and some related existing results.

A. Algebraic Graph Theory

An undirected graph $\mathcal{G} = (V, E)$ consists of a set of nodes V and a set of unordered edges E. An edge connecting V_i,$V_j \in V$ is denoted as $(i, j) \in E$. The adjacency matrix $\boldsymbol{A} = [a_{ij}]$ is defined where a_{ij} is 1 if $(i, j) \in E$ where $i \neq j$ and 0 otherwise. The degree matrix $\boldsymbol{D} = [d_{ii}]$ is a diagonal matrix with diagonal elements equal to the number of edges connected to the nodes. The Laplacian matrix $\boldsymbol{L} = [l_{ij}]$ is defined as $\boldsymbol{D} - \boldsymbol{A}$. The incidence matrix $\boldsymbol{B} = [b_{ik}]$ is defined where b_{ik} is 1 if V_i is the destination of edge-k, -1 if V_i is the source of edge-k and 0 otherwise. $|\boldsymbol{B}|$ is the entry-wise absolute value of $\boldsymbol{B}$. A well-known property is that $\boldsymbol{L} = \boldsymbol{B}\boldsymbol{B}^T$. Here, we denote $\boldsymbol{1}$ as the ones vector and it lies in the null space of $\boldsymbol{L}$ and $\boldsymbol{B}^T$. $\boldsymbol{I}_N$ is N-dimensional identity matrix. The undirected communication graph is comprised of nodes which represent agents and edges which represent the information exchange. Define N_k as the set of nodes with edge connected to agent-k, *i.e.*, the neighbors of agent-k, and $|N_k|$ as the total number of neighbors of agent-k.

B. Some Existing Results

The heading control dynamic model, which is frequently used in the literature, *e.g.,* [9], [10], [13], is shown by

$$\begin{aligned} \dot{\boldsymbol{r}}_k &= v_0[\cos\varphi_k, \sin\varphi_k]^T \\ \dot{\varphi}_k &= u_k, \end{aligned} \tag{1}$$

for $k = 1, 2, ..., N$, where $\boldsymbol{r}_k = [x_k, y_k]^T \in \mathbb{R}^2$ is the position, $\varphi_k \in \mathbb{R}$ is the heading angle, $v_0 \in \mathbb{R}$ is the velocity, and $u_k \in \mathbb{R}$ is the heading control input which controls the moving direction. Here the subindex k refers to agent-k. Model (1) is controlled by translation velocity v_0 and angular velocity $\dot{\varphi}_k$, and hence is also called the physical model.

In [9], they consider the problem that steering the N agents with dynamics (1) to form an evenly distributed circle, and also make the agents rotate around the centroid at a given angular velocity ϖ_0. Moreover, they further require the centroid to track a predefined center trajectory $\boldsymbol{r}_d(t) \in \mathbb{R}^2$. To achieve this additional goal, the authors propose the modified model (with constant $\bar{v}_0$):

$$\begin{aligned} \dot{\boldsymbol{r}}_k &= \bar{v}_0\left[\cos\theta_k, \sin\theta_k\right]^T + \dot{\boldsymbol{r}}_d \\ \dot{\theta}_k &= \bar{u}_k \end{aligned} \tag{2}$$

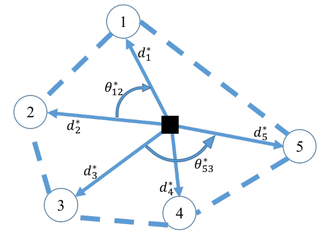

Fig. 1. The desired formation shape is given in dash line with counterclockwise order: 1-2-3-4-5. The square is the centroid. d_k^* is the length of agent-k's radius vector (arrowed line). θ_{kj}^* is the signed relative angle between radius vectors of agent-k and j, *e.g.*, $\theta_{12}^* < 0$ and $\theta_{53}^* > 0$.

where θ_k and $\bar{u}_k$ are the heading angle and the control input of (2), respectively. Note that the physical model is (1). Thus, after designing on (2), one needs to find the relation between the physical and the modified model, that is, $\dot{\boldsymbol{r}}_k = v_0\left[\cos\varphi_k, \sin\varphi_k\right]^T = \bar{v}_0\left[\cos\theta_k, \sin\theta_k\right]^T + \dot{\boldsymbol{r}}_d$. Then, the physical control law v_0 and u_k in (1) can be obtained by the following relations as in [9]:

$$\begin{aligned} v_0 &= |\dot{\boldsymbol{r}}_k| = \left|\bar{v}_0\left[\cos\theta_k, \sin\theta_k\right]^T + \dot{\boldsymbol{r}}_d\right| \\ u_k &= \frac{\ddot{\boldsymbol{r}}_k^T \boldsymbol{R}_{\frac{\pi}{2}} \dot{\boldsymbol{r}}_k}{|\dot{\boldsymbol{r}}_k|^2} \end{aligned} \tag{3}$$

where $\boldsymbol{R}_{\frac{\pi}{2}}$ is the $90°-$counterclockwise rotation matrix. In this paper, we propose an extended model to achieve more functionalities. Once the control law of the extended model is decided, the physical control law for model (1) can be obtained by similar derivations as in (3).

III. MODEL EXTENSION AND PROBLEM FORMULATION

Inspired by (2), we introduce our extended model which additionally considers arbitrary formation shape and transformation with an affine transformation command. Then, the considered problem is restated mathematically.

A. Extended Model

In [12], the authors consider ideal circular motion around origin, $\dot{\hat{\boldsymbol{r}}}_k = \varpi_0 \boldsymbol{R}_{\frac{\pi}{2}} \hat{\boldsymbol{r}}_k$, as the *virtual structure* to derive the extended model (2), where $\hat{\boldsymbol{r}}_k$ is the position of agent-k in the virtual structure and $\boldsymbol{r}_k^*$ is its desired actual position. Here, we first consider the unit circular motion whose center follows a given trajectory $\boldsymbol{r}_d$ as a virtual structure,

$$\dot{\hat{\boldsymbol{r}}}_k - \dot{\boldsymbol{r}}_d = \varpi_0 \boldsymbol{R}_{\frac{\pi}{2}}(\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d). \tag{4}$$

Then, to derive our extended model based on (4), we introduce a novel description of the desired formation shape with rotation-invariant property which is suitable for rotating formation. Given any desired formation shape, say, Fig. 1. First, calculate the centroid which the desired formation shape rotates around. Then, let the vectors from the centroid to each agent be the radius vectors, and define d_k^* as the length of it and θ_{kj}^* as the desired relative heading phase (angle) between agent-k's and agent-j's, where the term *phase* implies sign sensitive, *i.e.*, $\theta_{kj}^* = -\theta_{jk}^*$, see Fig. 1.

By the descriptions, for a desired formation with rotation where agent-k is supposed to rotate around $\boldsymbol{r}_d$ with the radius d_k^*, the actual distance from agent-k to $\boldsymbol{r}_d$ should be scaled by a factor d_k^* with respect to the distance from $\hat{\boldsymbol{r}}_k$ to $\boldsymbol{r}_d$ in virtual structure (4), *i.e.*, $\boldsymbol{r}_k^* - \boldsymbol{r}_d = d_k^*(\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d)$. Moreover, let $\theta_k^*(t)$ be the desired heading angle of agent-k which satisfies $\theta_k^*(t) - \theta_j^*(t) = \theta_{kj}^*$ and $\dot{\theta}_k^* = \varpi_0$, $\forall k, j = 1, \ldots, N$. Then, the unit vector $\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d$ can be expressed in polar coordinates, *i.e.*, $\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d = [\sin\theta_k^*, -\cos\theta_k^*]^T$. If we further consider a *reference affine transformation command* $\boldsymbol{G}^*$, which is a 2-by-2 matrix, such as scaling, rotating, or shearing, then the desired relation between $\boldsymbol{r}_k^*$ and $\hat{\boldsymbol{r}}_k$ is

$$\boldsymbol{r}_k^* - \boldsymbol{r}_d = d_k^* \boldsymbol{G}^* (\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d) = d_k^* \boldsymbol{G}^* [\sin\theta_k^*, -\cos\theta_k^*]^T. \quad (5)$$

Differentiating (5) we obtain $\dot{\boldsymbol{r}}_k^* - \dot{\boldsymbol{r}}_d = d_k^* \dot{\boldsymbol{G}}^* (\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d) + d_k^* \boldsymbol{G}^* (\dot{\hat{\boldsymbol{r}}}_k - \dot{\boldsymbol{r}}_d)$. By virtual structure (4), we have $\dot{\boldsymbol{r}}_k^* - \dot{\boldsymbol{r}}_d = d_k^*(\varpi_0 \boldsymbol{G}^* - \dot{\boldsymbol{G}}^* \boldsymbol{R}_{\frac{\pi}{2}}) \boldsymbol{R}_{\frac{\pi}{2}} (\hat{\boldsymbol{r}}_k - \boldsymbol{r}_d)$, which is equivalent to

$$\dot{\boldsymbol{r}}_k^* - \dot{\boldsymbol{r}}_d = d_k^*(\varpi_0 \boldsymbol{G}^* - \dot{\boldsymbol{G}}^* \boldsymbol{R}_{\frac{\pi}{2}}) \left[\cos\theta_k^*, \sin\theta_k^*\right]^T. \quad (6)$$

Now, the desired position and desired rotational motion of agent-k are (5) and (6), respectively. As a result, the extended model for N numbered agents is proposed based on (6):

$$\begin{aligned}
\dot{\boldsymbol{r}}_k &= d_k^*(\varpi_0 \boldsymbol{G}_k - \dot{\boldsymbol{G}}_k \boldsymbol{R}_{\frac{\pi}{2}}) \left[\cos\theta_k, \sin\theta_k\right]^T + \boldsymbol{v}_k \\
\dot{\boldsymbol{v}}_k &= \boldsymbol{\tau}_k \\
\ddot{\boldsymbol{G}}_k &= \boldsymbol{T}_k \\
\dot{\theta}_k &= \bar{u}_k \qquad \text{for } \ k = 1, 2, \ldots, N
\end{aligned} \quad (7)$$

where θ_k is the heading angle of agent-k, $\boldsymbol{\tau}_k \in \mathbb{R}^2$ is the translation control that steers $\boldsymbol{v}_k$ to follow the given $\dot{\boldsymbol{r}}_d$, $\boldsymbol{T}_k \in \mathbb{R}^{2\times 2}$ is the affine transformation command control such that $\boldsymbol{G}_k$ tracks the reference $\boldsymbol{G}^*(t)$, and $\bar{u}_k \in \mathbb{R}$ is the heading control which is in charge of the moving direction.

Remark 1. Suppose that i) affine transformation is not considered, *i.e.*, $\boldsymbol{G}_k = \boldsymbol{I}_2$, ii) all agents receive $\dot{\boldsymbol{r}}_d$ directly, *i.e.*, $\boldsymbol{v}_k$ can be replaced by $\dot{\boldsymbol{r}}_d$, and iii) consider the circular formation, *i.e.*, d_k^* is the circle radius and $\bar{v}_0 = \varpi_0 d_k^*$, then our extended model (7) degenerates exactly into (2).

Remark 2. In [10] and [12], $\boldsymbol{G}^*$ and $\boldsymbol{r}_d$ are assumed to be globally accessible in the common global frame and this is sometimes restrictive. In this paper, $\boldsymbol{G}_k$ and $\boldsymbol{v}_k$ is controlled to track the reference $\boldsymbol{G}^*$ and $\dot{\boldsymbol{r}}_d$ through communication links instead of assuming globally receivable. Moreover, the requirement of a common global frame can be relieved.

B. Problem Formulation with the Extended Model

With the extended model proposed in (7), we are able to re-formulate the ordered formation control problem. Given the desired smooth center trajectory $\boldsymbol{r}_d(t) \in \mathbb{R}^2$, the desired formation shape in terms of θ_{kj}^* and d_k^*, the smooth reference affine transformation command $\boldsymbol{G}^*(t)$, and the constant angular velocity ϖ_0. The communications in the MAS is represented by the undirected connected communication graph $\mathcal{G}$. Moreover, at least one agent can receive $\boldsymbol{r}_d$ and $\boldsymbol{G}^*$.In addition, the information should be measured locally from the neighbors and in the local frame of each agent.

As discussed in Section II, the main objective is to design the control laws, $\boldsymbol{\tau}_k$, $\boldsymbol{T}_k$ and $\bar{u}_k$, such that the numbered MAS with extended dynamics model (7) can (i) form into the arbitrarily given shape with specified order relation, (ii) rotate around the centroid at the angular velocity ϖ_0, (iii) keep the centroid track the given trajectory $\boldsymbol{r}_d(t)$, and (iv) transform into various formation shapes by the reference affine transformation command $\boldsymbol{G}^*(t)$. Mathematically, we want to design the control such that

$$\boldsymbol{G}_k d_k^* \left[\sin\theta_k, -\cos\theta_k\right]^T - (\boldsymbol{r}_k - \boldsymbol{r}_d) \to 0 \quad (8)$$

$$\theta_{kj} \to \theta_{kj}^* \quad (9)$$

$$\bar{u}_k \to \varpi_0 \quad (10)$$

$$\boldsymbol{v}_k \to \dot{\boldsymbol{r}}_d (= \boldsymbol{v}_d) \quad (11)$$

$$\boldsymbol{G}_k \to \boldsymbol{G}^*,\ \dot{\boldsymbol{G}}_k \to \dot{\boldsymbol{G}}^* \quad (12)$$

where $\theta_{kj} := \theta_k - \theta_j$ is the relative heading phase between agent-k and j. Note that (8)-(12) implies that the desired position (5) and desired rotational motion (6) are achieved.

IV. MAIN RESULTS

In this section, we first introduce the *phase penalty flow exchanging mechanism* to facilitate the order issue. Then, the control law is proposed with stability analysis.

A. Phase Penalty Flow Exchanging Mechanism

Given the connected communication graph as in subsection III-B. Consider the agent-k which aims to steer $\theta_{kj} \to \theta_{kj}^*, \forall j \in N_k$. Define $\tilde{\theta}_{kj} = \theta_{kj} - \theta_{kj}^*$ as the relative heading phase error between agent-k and j, and the *phase penalty* of agent-k as $\zeta_k = \sum_{j \in N_k} (1 - \cos(\tilde{\theta}_{kj})) \geq 0$. Let $w_k(t)$ be the weighting parameter, which is lower bounded by an arbitrarily chosen positive constant $\underline{w}_k$, to express the weight of the agent-k's phase penalty, and the value of w_k will be designed by an update law later. The reason why not using constant w_k will be remarked after the main theorem. Define the *phase penalty flow* of agent k as $\Phi_k = (w_k - \underline{w}_k)\zeta_k \geq 0$. The term *flow* implies the fluidity. More precisely, the agent-k can arbitrarily distribute its Φ_k to neighbors and the value that agent-k distributes to its neighbor agent-j is denoted as $\phi_{kj} \geq 0$. This can be realized by updating w_k, that is, if w_k increases based on the net flow of agent-k, then the Φ_k raises as receiving more penalty flow from the neighbors. As a result, we first derive the net flow change and then propose the update law of w_k based on the net flow of agent-k.

As a design tool, we make agent-k distribute all its phase penalty flow to the neighbors, that is, $\Phi_k = \sum_{j \in N_k} \phi_{kj}$. This can be implemented by randomly partitioning the phase penalty flow Φ_k into $|N_k|$ parts. Note that ϕ_{kj} and ϕ_{jk} are not necessarily equal and in fact usually different. In addition, once $\zeta_k = 0$, which means that the agent-k is in a correct order with its neighbors, it should stop receiving penalty flow from its neighbors and hence can be removed from the flow exchanging mechanism at the moment. By the above distribution process, the net flow of agent-k after an iteration is $-\Phi_k + \sum_{j \in N_k} \phi_{jk}$, which includes flowing out Φ_k and receiving the penalty flow from its neighbors. Since the routed out flows are received by the neighbors, the total net flow of the MAS equals to 0, *i.e.*,

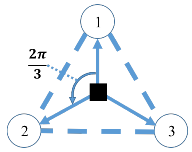

Fig. 2. Suppose the desired formation shape is regular triangle, arbitrarily select an agent to be at 0 degree, say, agent-1. Then we have agent-2 at $\frac{2\pi}{3}$ and agent-3 at $\frac{4\pi}{3}$. Stack them into the reference heading vector $\boldsymbol{\theta}^r = [0, \frac{2\pi}{3}, \frac{4\pi}{3}]^T$. Note that $\theta_2^r - \theta_1^r = \theta_{21}^* = \frac{2\pi}{3}$ and so on.

$$\sum_{k=1}^{N}\Big(-\Phi_k + \sum_{j\in N_k} \phi_{jk}\Big) = 0 \tag{13}$$

Now, consider a fixed formation shape that undergoes the flow exchanging mechanism. Because the formation shape remains unchanged, no matter how the agents share or distribute the penalty flow by updating $w_k, \forall k = 1, \ldots, N$, the total penalty flow should be the same, that is, $\sum_{k=1}^{N} \dot{\Phi}_k = 0$. By $\dot{\zeta}_k = 0$ due to the unchanged formation shape, we have $\sum_{k=1}^{N} \dot{\Phi}_k = \sum_{k=1}^{N} \dot{w}_k \zeta_k$. Thus, we need to design an update law of w_k based on the net flow of agent-k which satisfies $\sum_{k=1}^{N} \dot{w}_k \zeta_k = 0$. This goal is proved together with (13) and the following designed update law:

$$\dot{w}_k = \begin{cases} \frac{\delta}{\zeta_k}\left\{ -\Phi_k + \sum_{j\in N_k} \phi_{jk} \right\} & \text{, if } \zeta_k \neq 0 \\ 0 & \text{, if } \zeta_k = 0 \end{cases} \tag{14}$$

for $k = 1, \ldots, N$. Here δ is a positive constant that can be arbitrarily chosen. Recall that $w_k(t)$ is claimed to be lower bounded by $\underline{w}_k$. This can be verified by the fact that once w_k decreases to $\underline{w}_k$, then $\Phi_k = (w_k - \underline{w}_k)\zeta_k$ becomes 0 and leads to $\dot{w}_k \geq 0$ by (14).

B. Notations used in theorem and proof

We introduce some variables to present the following main theorem and its proof. Construct the *reference heading vector*, $\boldsymbol{\theta}^r \in \mathbb{R}^N$, by the following procedure: Given a desired formation shape, select an agent to be at 0°, then the corresponding degrees of the other agents are naturally determined by the desired relative heading phase. Denote agent-k's corresponding degree as θ_k^r and stack all into a vector $\boldsymbol{\theta}^r$. One can check that $\theta_k^r - \theta_j^r = \theta_{kj}^*$, see Fig. 2.

With the knowledge of $\boldsymbol{\theta}^r$, we define the *heading shift* $\hat{\boldsymbol{\theta}} := [\theta_1 - \theta_1^r, \ldots, \theta_N - \theta_N^r]^T = \boldsymbol{\theta} - \boldsymbol{\theta}^r \in \mathbb{R}^N$ with the property: $\tilde{\theta}_{kj} = \theta_{kj} - \theta_{kj}^* = \theta_k - \theta_j - (\theta_k^r - \theta_j^r) = \hat{\theta}_k - \hat{\theta}_j$.

Define the left-hand side of (8) as the rotational motion error $\boldsymbol{e}_{kd}$ of agent-k. Then, $\boldsymbol{e}_{kj} := \boldsymbol{e}_{kd} - \boldsymbol{e}_{jd}$ is the *relative rotational motion error* which does not need $\boldsymbol{r}_d$. Let $\boldsymbol{v}_{kj} = \boldsymbol{v}_k - \boldsymbol{v}_j$ and $\boldsymbol{G}_{kj} = \boldsymbol{G}_k - \boldsymbol{G}_j$ be the relative errors. Denote $\boldsymbol{E}_d = [\boldsymbol{e}_{1d}^T \ldots \boldsymbol{e}_{kd}^T \ldots \boldsymbol{e}_{Nd}^T]^T \in \mathbb{R}^{2N}$ as the stacked rotational error, $\boldsymbol{w} = [w_1 \ldots w_k \ldots w_N]^T \in \mathbb{R}^N$ as the weighting vector, and $\tilde{\boldsymbol{v}} = [\tilde{\boldsymbol{v}}_1^T \ldots \tilde{\boldsymbol{v}}_k^T \ldots \tilde{\boldsymbol{v}}_N^T]^T \in \mathbb{R}^{2N}$ as the stacked translation error where $\tilde{\boldsymbol{v}}_k = \boldsymbol{v}_k - \boldsymbol{v}_d$, for $k = 1, 2, ..., N$.

For consistence, recall that N_k denotes the set of the neighbors of agent-k. Here, we define $\bar{N}_k$ as the extended set which additionally considers whether the reference signal is accessible to agent-k or not. To represent the connection between agent-k and the reference signal, we define a diagonal matrix $\boldsymbol{P} = [p_{ii}] \in \mathbb{R}^{N\times N}$ where p_{ii} is 1 if agent-i has access to the reference and 0 otherwise. Let $\bar{\boldsymbol{L}} := (\boldsymbol{L} + \boldsymbol{P}) \otimes \boldsymbol{I}_2 \in \mathbb{R}^{2N\times 2N}$ be the extended augmented Laplacian matrix where $\otimes$ denotes Kronecker product. Since $\boldsymbol{P}$ has at least one positive entry, $\bar{\boldsymbol{L}}$ is positive definite.

C. Control Law Derivation and Stability Analysis

To derive the control laws, $\bar{u}_k$, $\boldsymbol{\tau}_k$, and $\boldsymbol{T}_k$, a lemma from [2] is first given, which will facilitate the design of $\boldsymbol{T}_k$.

Lemma 1. ([2]) Consider agents with second-order dynamics given by $\ddot{\xi}_i = u_i$, where u_i is control input. Besides, exogenous reference signal satisfies the dynamics: $\ddot{\xi}^r = f(t, \xi^r, \dot{\xi}^r)$, where $f(\cdot,\cdot,\cdot)$ is piecewise continuous in t and locally Lipschitz in ξ^r, $\dot{\xi}^r$. The control law $u_i = \frac{1}{\eta_i}\sum_{j=0}^{N} a_{ij}\left[u_j - K_1(\xi_i - \xi_j) - K_2(\dot{\xi}_i - \dot{\xi}_j)\right]$ makes all agents follow the reference signal ξ^r. Where $K_i > 0$, a_{ij} are adjacency matrix elements and $\eta_i = \sum_{j=0}^{N} a_{ij}$ where a_{i0} is 1 if agent-i has access to ξ^r and 0 otherwise. $\xi_0 = \xi^r$, $\dot{\xi}_0 = \dot{\xi}^r$ and $u_0 = f(t, \xi^r, \dot{\xi}^r)$.

Our main results are shown in the following theorem where we first propose the control law in terms of the global frame, and then show the equivalence in the local frames.

Theorem 1. Consider the MAS (7) with randomly given initial positions. Suppose $\mathcal{G}$ is connected and at least one agent can receive $\boldsymbol{r}_d(t)$, $\boldsymbol{G}^*(t)$ and their first and second derivatives. With the update law of w_k in (14), if the control law $\bar{u}_k$, $\boldsymbol{\tau}_k$, and $\boldsymbol{T}_k$ is designed as

$$\bar{u}_k = \varpi_0 - \{\alpha \boldsymbol{h}_k^T(\sum_{j\in\bar{N}_k} \boldsymbol{e}_{kj}) + \beta \sum_{j\in N_k}(w_k + w_j)\sin(\tilde{\theta}_{kj})\} \tag{15}$$

$$\boldsymbol{\tau}_k = \frac{1}{|\bar{N}_k|}(\sum_{j\in\bar{N}_k} \dot{\boldsymbol{v}}_j) - \frac{1}{|\bar{N}_k|}\sum_{j\in\bar{N}_k}(\gamma \boldsymbol{v}_{kj} - \alpha \boldsymbol{e}_{kj}) \tag{16}$$

$$\boldsymbol{T}_k = \frac{1}{|\bar{N}_k|}(\sum_{j\in\bar{N}_k} \ddot{\boldsymbol{G}}_j) - \frac{1}{|\bar{N}_k|}\sum_{j\in\bar{N}_k}(\lambda \dot{\boldsymbol{G}}_{kj} + \mu \boldsymbol{G}_{kj}) \tag{17}$$

for $k = 1, \ldots, N$, where $\boldsymbol{h}_k = d_k^* \boldsymbol{G}_k [\cos\theta_k, \sin\theta_k]^T$ and $\alpha, \beta, \gamma, \lambda, \mu$, are arbitrarily chosen positive parameters. Then, the MAS achieves (8)-(12) asymptotically.

Before proving Theorem 1, we will first show that the control law (15)-(17) can be implemented in agents' local frames. Suppose agent-k has a local frame with counter clockwise rotation angle ψ_k relative to the common global frame. Let variables superscripted by k be the variables with respect to agent-k's local frame and $\boldsymbol{R}(\theta)$ be the rotation matrix of angle θ. Then, we have the relations for vectors: $\boldsymbol{e}_{kj}^k = \boldsymbol{R}(-\psi_k)\boldsymbol{e}_{kj}$, $\boldsymbol{v}_{kj}^k = \boldsymbol{R}(-\psi_k)\boldsymbol{v}_{kj}$, $\boldsymbol{\tau}_k^k = \boldsymbol{R}(-\psi_k)\boldsymbol{\tau}_k$, $[\cos\theta_k^k, \sin\theta_k^k]^T = \boldsymbol{R}(-\psi_k)[\cos\theta_k, \sin\theta_k]^T$, and for matrix: $\boldsymbol{G}_k^k = \boldsymbol{R}(-\psi_k)\boldsymbol{G}_k\boldsymbol{R}(\psi_k)$, $\boldsymbol{T}_k^k = \boldsymbol{R}(-\psi_k)\boldsymbol{T}_k\boldsymbol{R}(\psi_k)$. Note that $\bar{u}_k^k = \bar{u}_k$ as it is a scalar. Recall $\boldsymbol{h}_k$, and define $\boldsymbol{h}_k^k = d_k^* \boldsymbol{G}_k^k [\cos\theta_k^k, \sin\theta_k^k]^T$ which is obtainable in agent-k's local frame. Now, we prove that $\boldsymbol{h}_k^k = \boldsymbol{R}(-\psi_k)\boldsymbol{h}_k$, by the following relations: $\boldsymbol{h}_k^k = d_k^* \boldsymbol{G}_k^k [\cos\theta_k^k, \sin\theta_k^k]^T = d_k^* \boldsymbol{R}(-\psi_k)\boldsymbol{R}(\psi_k)\boldsymbol{G}_k^k\boldsymbol{R}(-\psi_k)[\cos\theta_k, \sin\theta_k]^T = \boldsymbol{R}(-\psi_k)\boldsymbol{h}_k$. Combine with the relation, $\boldsymbol{h}_k^T(\sum_{j\in\bar{N}_k} \boldsymbol{e}_{kj}) =$

$(\boldsymbol{R}(-\psi_k)\boldsymbol{h}_k)^T(\boldsymbol{R}(-\psi_k)\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}) = \boldsymbol{h}_k^{k^T}(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}^k)$, then we have (15) be equivalent to

$$\bar{u}_k^k = \varpi_0 - \{\alpha\boldsymbol{h}_k^{k^T}(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}^k) + \beta\sum_{j\in N_k}(w_k+w_j)\sin(\tilde{\theta}_{kj})\} \tag{18}$$

Multiply $\boldsymbol{R}(-\psi_k)$ to both sides of (16) and we have

$$\boldsymbol{\tau}_k^k = \frac{1}{|\bar{N}_k|}\sum_{j\in\bar{N}_k}\left(\dot{\boldsymbol{v}}_j^k - \gamma\boldsymbol{v}_{kj}^k + \alpha\boldsymbol{e}_{kj}^k\right) \tag{19}$$

Pre-multiply $\boldsymbol{R}(-\psi_k)$ and post-multiply $\boldsymbol{R}(\psi_k)$ to both sides of (17), then we have

$$\boldsymbol{T}_k^k = \frac{1}{|\bar{N}_k|}\sum_{j\in\bar{N}_k}\left(\ddot{\boldsymbol{G}}_j^k - \lambda\dot{\boldsymbol{G}}_{kj}^k - \mu\boldsymbol{G}_{kj}^k\right) \tag{20}$$

That is, (15)-(17) is equivalent to (18)-(20) where the control law is implemented in agents' local frames. Thus, here we give Theorem 1 and its proof in terms of the global frame, while in practice the control law is implemented with (18)-(20) in each agent's local frame.

Proof. Choose the Lyapunov Candidate Function

$$V = \frac{\alpha}{2}\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\boldsymbol{E}_d + \beta\boldsymbol{w}^T|\boldsymbol{B}|(\boldsymbol{1}-\cos(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})) + \frac{1}{2}\tilde{\boldsymbol{v}}^T\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}} \tag{21}$$

where the $\cos(\cdot)$ is element-wise. The first term indicates the error between the centroid and $\boldsymbol{r}_d$. The second term penalizes the relative phase error to achieve ordered formation. And the last term is for velocity tracking. Thus, we will first show that V monotonically decreases to 0 and this proves (8-11). Then, (12) will be proved by using Lemma 1 later on.

The first and the third term of V are in quadratic form and the second term is constructed with all positive elements. As a result, $V\geq 0$ is ensured. Then, to prove the monotonicity of V, differentiate $\boldsymbol{e}_{kd}$ to assist the derivation of $\dot{V}$: $\dot{\boldsymbol{e}}_{kd} = (\dot{\theta}_k - \varpi_0)d_k^*\boldsymbol{G}_k[\cos\theta_k, \sin\theta_k]^T - (\boldsymbol{v}_k - \boldsymbol{v}_d)$. Recall that $\boldsymbol{h}_k = d_k^*\boldsymbol{G}_k[\cos\theta_k,\sin\theta_k]^T$, and define:

$$\boldsymbol{H} = \begin{bmatrix}\boldsymbol{h}_1 & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & \boldsymbol{h}_N\end{bmatrix}\in\mathbb{R}^{2N\times N}$$

Then, derive $\dot{V}$ as follows:

$$\begin{aligned}\dot{V} &= \alpha\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\dot{\boldsymbol{E}}_d + \beta\boldsymbol{w}^T|\boldsymbol{B}|(\sin(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})\circ(\boldsymbol{B}^T\dot{\hat{\boldsymbol{\theta}}}))\\ &\quad + \beta\dot{\boldsymbol{w}}^T|\boldsymbol{B}|(\boldsymbol{1}-\cos(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})) + \tilde{\boldsymbol{v}}^T\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}} \qquad (22)\\ \dot{V} &= \alpha\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\boldsymbol{H}(\dot{\boldsymbol{\theta}}-\varpi_0\boldsymbol{1}) + \beta\dot{\boldsymbol{w}}^T|\boldsymbol{B}|(\boldsymbol{1}-\cos(\boldsymbol{B}^T\hat{\boldsymbol{\theta}}))\\ &\quad + \beta\sin(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})^T\mathbb{D}\boldsymbol{B}^T\dot{\hat{\boldsymbol{\theta}}} - \alpha\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}} + (\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}})^T\tilde{\boldsymbol{v}} \qquad (23)\\ \dot{V} &= \{\alpha\boldsymbol{E}_d^T\bar{\boldsymbol{L}}\boldsymbol{H} + \beta\sin(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})^T\mathbb{D}\boldsymbol{B}^T\}(\dot{\boldsymbol{\theta}}-\varpi_0\boldsymbol{1})\\ &\quad + \{\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}} - \alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d\}^T\tilde{\boldsymbol{v}} + \beta\dot{\boldsymbol{w}}^T|\boldsymbol{B}|(\boldsymbol{1}-\cos(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})) \qquad (24)\end{aligned}$$

where $\mathbb{D}$ is diagonal matrix with $\mathbb{D}_{ii} = (|\boldsymbol{B}|^T\boldsymbol{w})_i$, and $\circ$ denotes Hadamard product. Note that from (23) to (24), we use the fact that $\boldsymbol{1}$ is in the null space of $\boldsymbol{B}^T$.

Represent the right-hand side of (24) by $\dot{V}_1 + \dot{V}_2 + \dot{V}_3$, which are related to heading control, translation velocity control, and weighting update law, respectively. In the following, we will design $\bar{u}_k$ and $\boldsymbol{\tau}_k$ to make $\dot{V}_1\leq 0$ and $\dot{V}_2\leq 0$, respectively, while $\dot{V}_3$ will be shown equal to 0.

To make $\dot{V}_1\leq 0$, the heading control is selected as:

$$\dot{\boldsymbol{\theta}} = \varpi_0\boldsymbol{1} - \{\alpha\boldsymbol{H}^T\bar{L}\boldsymbol{E}_d + \beta\boldsymbol{B}\mathbb{D}\sin(\boldsymbol{B}^T\hat{\boldsymbol{\theta}})\} \tag{25}$$

Expand it and we have the heading control (15), where the second term is for ensuring desired rotational motion and the third term is for desired relative heading phase $\theta_{kj}^*, \forall j\in N_k$.

Consider $\dot{V}_2$, if we make the bracket part in it equal to $(-\gamma\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}})^T$, then $\dot{V}_2 = -\gamma\tilde{\boldsymbol{v}}^T\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}}\leq 0$. Therefore, we design

$$\bar{\boldsymbol{L}}\dot{\tilde{\boldsymbol{v}}} = \alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d - \gamma\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}}. \tag{26}$$

It can be rearranged into $((\boldsymbol{D}+\boldsymbol{P})\otimes\boldsymbol{I}_2)\dot{\tilde{\boldsymbol{v}}} = (\boldsymbol{A}\otimes\boldsymbol{I}_2)\tilde{\boldsymbol{v}} + (\alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d - \gamma\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}})$. $\boldsymbol{D}+\boldsymbol{P}$ is invertible, and by $(A\otimes B)^{-1} = A^{-1}\otimes B^{-1}$, $(A\otimes B)(C\otimes D) = (A\otimes C)(B\otimes D)$, we get: $\dot{\tilde{\boldsymbol{v}}} = ((\boldsymbol{D}+\boldsymbol{P})^{-1}\boldsymbol{A}\otimes\boldsymbol{I}_2)\tilde{\boldsymbol{v}} + ((\boldsymbol{D}+\boldsymbol{P})^{-1}\otimes\boldsymbol{I}_2)(\alpha\bar{\boldsymbol{L}}\boldsymbol{E}_d - \gamma\bar{\boldsymbol{L}}\tilde{\boldsymbol{v}})$. Expand it and move the $\dot{\boldsymbol{v}}_d$ term in $\dot{\tilde{\boldsymbol{v}}}$ to the right-hand side of the equation, then we obtain the translation velocity control (16), where the first part is for acceleration consensus, while the second part is for velocity consensus. Once all $\boldsymbol{e}_{kj}$ converge to $\boldsymbol{0}$, the control law becomes the consensus algorithm with time-varying reference as in [1].

The last part, $\dot{V}_3$ can be expanded and expressed as $\beta\sum_{k=1}^N\dot{w}_k\zeta_k$. By (14), we get $\dot{V}_3 = 0$. To sum up, $V\geq 0$ and $\dot{V} = -\sum_{k=1}^N\{\alpha\boldsymbol{h}_k^T(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}) + \beta\sum_{j\in N_k}(w_k + w_j)\sin(\tilde{\theta}_{kj})\}^2 - \gamma\tilde{\boldsymbol{v}}^T\bar{L}\tilde{\boldsymbol{v}}\leq 0$. As a result, by Lasalle Invariance Principle, it converges to the largest invariance set $\{\dot{V}=0\}$. More precisely,

$$\tilde{\boldsymbol{v}}_k = \boldsymbol{0} \tag{27}$$

$$\alpha\boldsymbol{h}_k^T(\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}) + \beta\sum_{j\in N_k}(w_k+w_j)\sin(\tilde{\theta}_{kj}) = 0 \tag{28}$$

for $k = 1,\dots,N$. (27) holds implies that $\boldsymbol{v}_k = \boldsymbol{v}_d$, and (28) holds implies that $\bar{u}_k = \varpi_0$ by (15). Then $\dot{\boldsymbol{e}}_{kd} = \boldsymbol{0}$ for $k = 1,\dots,N$ and also $\dot{\tilde{\theta}}_{kj} = 0, \forall k\neq j$. That is, $\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}$ for $k = 1,\dots,N$, and $\tilde{\theta}_{kj}, \forall k\neq j$ all converge to some constants. Particularly, the vector $\boldsymbol{h}_k$ keeps moving and the weighting w_k keeps updating; however, (28) maintains 0. That is, the only scenario is that $\sum_{j\in\bar{N}_k}\boldsymbol{e}_{kj}$ for $k = 1,\dots,N$ and $\tilde{\theta}_{kj}, \forall k\neq j$ all converge to the constant, 0. Moreover, $\bar{\boldsymbol{L}}\boldsymbol{E}_d = \boldsymbol{0}$ leads to $\boldsymbol{e}_{kd} = \boldsymbol{0}$ for $k = 1,\dots,N$. As a result, (8)-(11) are proved.

To achieve (12), a second order consensus control with a time-varying reference is required. Thus, the result of Lemma 2 can be directly applied to obtain the affine transformation command control, which is rearranged into (20). ∎

Remark 3. Suppose without the penalty flow exchanging mechanism and update law (14), *i.e.*, w_k are constants for all k. Then, by the same process, we can still have (28) but with *constant* w_k, which only ensures $\boldsymbol{e}_{kd} = 0$ and $\sum_{j\in N_k}(w_k + w_j)\sin(\tilde{\theta}_{kj}) = 0$, but $\tilde{\theta}_{kj} = 0$ is not guaranteed. For example, suppose $\sin(\tilde{\theta}_{12}) + \sin(\tilde{\theta}_{13}) = 0$ with $\tilde{\theta}_{12}$ and $\tilde{\theta}_{13}$ being constants. Then, there exist infinitely many solutions in addition to the solution $\tilde{\theta}_{12} = \tilde{\theta}_{13} = 0$, which leaves the ordered formation unaccomplished.

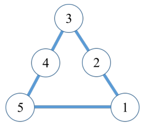

Fig. 3. Desired formation shape

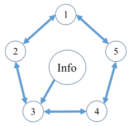

Fig. 4. Communication links

V. SIMULATION

In this section, an example of 5 agents MAS is illustrated. The desired formation shape and the communication links are given in Fig. 3 and Fig. 4, respectively, where d_k^* are $\frac{3\sqrt{116}}{5}$, $\frac{3\sqrt{26}}{5}$, $\frac{18}{5}$, $\frac{3\sqrt{26}}{5}$, $\frac{3\sqrt{116}}{5}$, the *reference heading vector* $\boldsymbol{\theta}^r$ is $[0, \pi - tan^{-1}5 - tan^{-1}\frac{5}{2}, \pi - tan^{-1}\frac{5}{2}, \pi + tan^{-1}5 - tan^{-1}\frac{5}{2}, 2\pi - 2tan^{-1}\frac{5}{2}]^T$, and the 'Info' in Fig. 4 means the information of $\boldsymbol{r}_d = [0.2t - 3 - 4\cos(0.03t + 0.15), 0.3t - 2 + 3\cos(0.06t + 2)]^T$ and $\boldsymbol{G}^* = (1 + 0.5\cos(0.05t))\boldsymbol{I}_2$ which adjusts the formation shape to pass through the valley in Fig. 5. The initial positions and headings are randomly given as $[6.6, -7.7]^T$, $[2.2, 2.3]^T$, $[3.6, -6.5]^T$, $[-4.8, 1]^T$, $[-4.2, -8.9]^T$ and 2, 1.6, -0.9, -1.9, -2.2, respectively. Then, by (18)-(20), the results are shown in Fig. 5. The MAS forms into the desired shape and its centroid tracks the given trajectory roughly at the fourth sampling time instant. Besides, the formation shape zooms out when passing through a small slit, and zooms in when the space is commodious. Moreover, the ordered formation is achieved as specified in Fig. 3.

To clearly demonstrate the ordered formation by our control law, we provide another simulation that the agents are initially all at the desired velocity and in desired formation shape, but with the wrong order 1-3-5-4-2, see Fig. 6. By our control law, the desired ordered formation 1-2-3-4-5 is achieved instead of sticking to the wrong order 1-3-5-4-2.

VI. CONCLUSION

In this paper, rotational formation with arbitrary shape and specified order is considered. The control law (18)-(20) measured locally from the neighbors and in each agent's local frame steers the MAS to achieve ordered formation, rotate around the centroid which tracks a desired trajectory, and transform the formation into various shapes to adapt to the environment by reference affine transformation command. We propose an extended model which additionally considers arbitrary formation shape and affine transformation compared with (2). Besides, the phase penalty flow exchanging mechanism is introduced to facilitate the success of ordered formation without imposing restrictions on initial condition as in [7], [13], or assuming specific communication graphs as in [16]. Unlike the existing results where $\boldsymbol{r}_d$ and $\boldsymbol{G}^*$

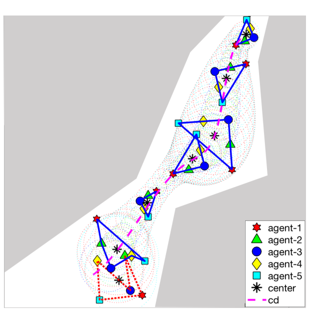

Fig. 5. Ordered formation which adapts to the environment

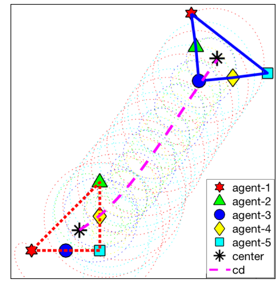

Fig. 6. Ordered formation is achieved by our design from the order relation 1-3-5-4-2 (dash line) to 1-2-3-4-5 (solid line).

are globally accessible, our design does not require global information in advance and preserves the ability of on-site changing. In the future work, a sphere model can be considered to extend the results to the spatial workspace.

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